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A GENERAL FRAMEWORK TO PROVIDE FOR THE  
OPTIMAL DISPATCH OF HYBRID RENEWABLE  
POWER SYSTEMS

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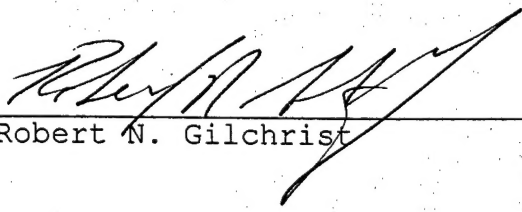
REPORT DOCUMENTATION PAGE			Form Approved OMB No. 0704-0188	
Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.				
1. AGENCY USE ONLY (Leave blank)		2. REPORT DATE 7.May.99		3. REPORT TYPE AND DATES COVERED DISSERTATION
4. TITLE AND SUBTITLE A GENERAL FRAMEWORK TO PROVIDE FOR THE OPTIMAL DISPATCH OF HYBRID RENEWABLE POWER SYSTEMS			5. FUNDING NUMBERS	
6. AUTHOR(S) MAJ GILCHRIST ROBERT N				
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) COLORADO SCHOOL OF MINES			8. PERFORMING ORGANIZATION REPORT NUMBER	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) THE DEPARTMENT OF THE AIR FORCE AFIT/CIA, BLDG 125 2950 P STREET WPAFB OH 45433			10. SPONSORING/MONITORING AGENCY REPORT NUMBER  FY99-95	
11. SUPPLEMENTARY NOTES				
12a. DISTRIBUTION AVAILABILITY STATEMENT Unlimited distribution In Accordance With AFI 35-205/AFIT Sup 1			12b. DISTRIBUTION CODE	
13. ABSTRACT (Maximum 200 words)				
14. SUBJECT TERMS			15. NUMBER OF PAGES 144	
			16. PRICE CODE	
17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT	

A thesis submitted to the Faculty and the Board of Trustees of the Colorado School of Mines in partial fulfillment of the requirements for the degree of Doctor of Philosophy (Mineral Economics).

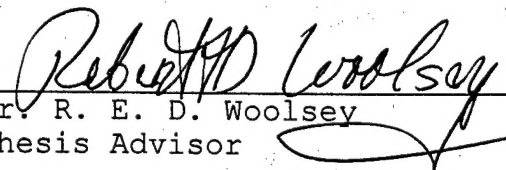
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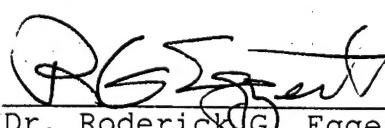
  
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## ABSTRACT

A significant number of people worldwide live in rural villages with only part-time electrical power or no power at all. In an effort to address this deficiency, the U.S. National Renewable Energy Laboratory (NREL) is developing hybrid renewable power systems composed of photovoltaic panels, wind turbines, battery banks, and diesel generators to deliver 24-hour power.

In keeping life cycle costs of these systems to a minimum, an appropriate dispatch strategy can contribute as much to reducing cost as the proper choice of system architecture. Current dispatch strategies consider only current net load and current state-of-charge of batteries. The framework developed here considers future net load realizations in determining optimal dispatch strategies.

The model is a Markov Decision Process solved using a Policy Iteration algorithm. The algorithm produces a policy that maps each state that the hybrid renewable power system can occupy to an optimal action. The dispatch policies



consider immediate and long run ramifications of an action on life-cycle-cost.

The results show that over different values of diesel fuel cost, battery wear cost, and loss of service penalties the optimal strategies minimize the long-run average cost. Furthermore, minimal implementation costs make the optimal strategies an attractive alternative to heuristic strategies that are currently used to dispatch hybrid renewable power systems.

## TABLE OF CONTENTS

	Page
ABSTRACT . . . . .	iii
LIST OF FIGURES . . . . .	vii
LIST OF TABLES . . . . .	x
ACKNOWLEDGMENTS. . . . .	xi
 Chapter	
1. INTRODUCTION. . . . .	1
1.1 Motivation. . . . .	1
1.2 Generic Hybrid Renewable Power Systems. . . . .	2
1.3 Problem Statement . . . . .	5
1.4 Objectives. . . . .	8
 2. REVIEW OF RELEVANT MODELS . . . . .	 11
2.1 Other HRPS Models . . . . .	11
2.2 Model Presented Here. . . . .	22
 3. MODELING THE HYBRID RENEWABLE POWER SYSTEM. . . . .	 27
3.1 Dynamic Programming . . . . .	27
3.2 Markov Decision Processes . . . . .	30
3.3 Model Specifics. . . . .	35
3.3.1 Stages. . . . .	35
3.3.2 State-Space . . . . .	36
3.3.3 Action Set. . . . .	40
3.3.4 Cost Structure. . . . .	41
3.3.5 Transition Probabilities. . . . .	44

Chapter	Page
3.4 Solution of the Model using Policy Iteration. . . . .	48
3.4.1 Policy Evaluation. . . . .	49
3.4.2 Policy Improvement . . . . .	51
3.5 Policy Iteration Algorithm . . . . .	52
3.6 Solution Procedures. . . . .	57
3.7 Evaluation of Optimal Strategies . . . . .	61
4. RESULTS AND ANALYSIS . . . . .	69
4.1 Simulation Results . . . . .	69
4.1.1 Analysis: Net Load Profile 1. . . . .	71
4.1.2 Analysis: Net Load Profile 2. . . . .	77
4.2 HOMER Results. . . . .	82
4.3 Implementation of the General Optimization Framework . . . . .	86
4.4 Conclusions. . . . .	93
5. ISSUES FOR FURTHER ANALYSIS. . . . .	97
REFERENCES CITED. . . . .	100
Appendix	
A. DESCRIPTION OF THE HYBRID OPTIMIZATION MODEL FOR ELECTRIC RENEWABLES (HOMER). . . . .	103
B. CODE FOR POLICY ITERATION ALGORITHM . . . . .	107
C. OPTIMAL DISPATCH CHARTS . . . . .	126

## LIST OF FIGURES

Figure	Page
1. Generic Hybrid Renewable Power System . . .	3
2. Comparison of HRPS Models . . . . .	26
3. State-Space. . . . .	37
4. Policy Iteration Algorithm (using iterative policy evaluation) for $V^*$ . . . .	53
5. Net Load Profile 1 . . . . .	57
6. Net Load Profile 2 . . . . .	59
7. Optimal Dispatch Decision Process . . . . .	66
8. HOMER Decision Process. . . . .	67
9. Optimal Actions, Net Load Profile 1, Scenario 1. . . . .	69
10. Optimal Action Profile, Net Load Profile 1, Scenario 1 . . . . .	70
11. Comparison of Dispatch Strategies, Net Load Profile 1. . . . .	72
12. Summary of Dispatch Strategy Results, Net Load Profile 1. . . . .	73
13. Quality of Service Analysis, Net Load Profile 1. . . . .	76
14. Comparison of Dispatch Strategies, Net Load Profile 2. . . . .	78

Figure		Page
15.	Summary of Dispatch Strategy Results, Net Load Profile 2 . . . . .	79
16.	Quality of Service Analysis, Net Load Profile 2. . . . .	81
17.	Optimal Strategy, Net Load Profile 1, Scenario 1 . . . . .	127
18.	Optimal Strategy, Net Load Profile 1, Scenario 2 . . . . .	128
19.	Optimal Strategy, Net Load Profile 1, Scenario 3 . . . . .	129
20.	Optimal Strategy, Net Load Profile 1, Scenario 4 . . . . .	130
21.	Optimal Strategy, Net Load Profile 1, Scenario 5 . . . . .	131
22.	Optimal Strategy, Net Load Profile 1, Scenario 6 . . . . .	132
23.	Optimal Strategy, Net Load Profile 1, Scenario 7 . . . . .	133
24.	Optimal Strategy, Net Load Profile 1, Scenario 8 . . . . .	134
25.	Optimal Strategy, Net Load Profile 1, Scenario 9 . . . . .	135
26.	Optimal Strategy, Net Load Profile 2, Scenario 1 . . . . .	136
27.	Optimal Strategy, Net Load Profile 2, Scenario 2 . . . . .	137

Figure		Page
28.	Optimal Strategy, Net Load Profile 2, Scenario 3 . . . . .	138
29.	Optimal Strategy, Net Load Profile 2, Scenario 4 . . . . .	139
30.	Optimal Strategy, Net Load Profile 2, Scenario 5 . . . . .	140
31.	Optimal Strategy, Net Load Profile 2, Scenario 6 . . . . .	141
32.	Optimal Strategy, Net Load Profile 2, Scenario 7 . . . . .	142
33.	Optimal Strategy, Net Load Profile 2, Scenario 8 . . . . .	143
34.	Optimal Strategy, Net Load Profile 2, Scenario 9 . . . . .	144

## LIST OF TABLES

Table	Page
1. Explanation of Terms (Kandil) . . . . .	14
2. Explanation of Terms (Barley) . . . . .	18
3. Explanation of Terms (Bakirtzis). . . . .	21
4. Typical Characteristics of a Dynamic Program . . . . .	28
5. Scenarios . . . . .	60
6. Results from HOMER. Net Load Profile 1 . .	84
7. Results from HOMER. Net Load Profile 2 . .	85
8. Optimal Strategies vs. Next Best Heuristic . . . . .	90
9. Benefit-Cost Analysis, Net Load Profile 1 . . . . .	92
10. Benefit-Cost Analysis, Net Load Profile 2 . . . . .	93

## ACKNOWLEDGMENTS

First and foremost, I wish to express my deepest gratitude to my friend and mentor Jeff Grobman who in more than one instance kept me from wandering aimlessly down the wrong path. I hope some day to repay him for the accurate and timely advice he provided me over the past two years.

Many thanks to Dr. R.E.D. Woolsey whose unique outlook on life and the Operations Research profession can be a lesson to us all about the power of a vision and the perseverance to carry it out.

I also wish to thank the following members of my committee: Dr. Robert Frost, Dr. William Navidi, and Dr. Roderick Eggert.

I would also like to acknowledge the help of Dr. Peter Lilienthal and Mr. Tom Lambert of the National Renewable Energy Laboratory. Peter planted the idea that got me started on this research effort and Tom provided me with encouragement and timely assistance with HOMER.



Of course, I cannot forget the support provided by my wife, Karen, who endured many lonely days and nights over the past few years. And last but not least, my son Will whose carefree attitude (what other kind does a two-year-old have?) kept me focused on what's really important in life.

## Chapter 1

### INTRODUCTION

#### 1.1 Motivation

Worldwide, some two billion people live without electricity (Lilienthal 1995, 1). These people who lack such things as clean water, refrigerated foods and medicines, telephones, and radios, miss the social, economic, and health benefits that these necessities afford the rest of the world. The goal of the U.S. National Renewable Energy Laboratory (NREL) is to integrate affordable and practical renewable energy technologies, such as photovoltaics (PV) and wind power, into rural economic development. By doing so, these renewable energy systems could provide many benefits, such as water pumping capability, electrification of schools, community centers and health clinics, and desalinization, or disinfection of water.

In addition to those people who do not have electricity, a sizable number of people worldwide live in rural villages with only part-time electrical service

provided through diesel generation. The Renewables for Sustainable Village Power group at NREL is developing hybrid renewable power systems (HRPS) that include small wind turbines, photovoltaic panels, battery storage, and diesel generators to provide 24-hour power to remote villages. These hybrid systems often outperform the generator-only systems with higher quality, greater reliability, lower cost, and reduced environmental degradation (Flowers 1997).

### 1.2 Generic Hybrid Renewable Power Systems

A generic HRPS is shown in figure 1. The "AC generators" represent various sources that produce alternating current. Such sources include AC wind turbines and AC diesel generators. It is the diesel generator that we are trying to incorporate into, or build a HRPS around, to provide 24-hour power. The "DC generators" represent various energy sources that produce direct current. Such devices include DC wind turbines, DC diesel generators and photovoltaic panels. "AC loads," represent typical fixtures or appliances that require alternating current similar to those that one might find in any U.S. household. "DC

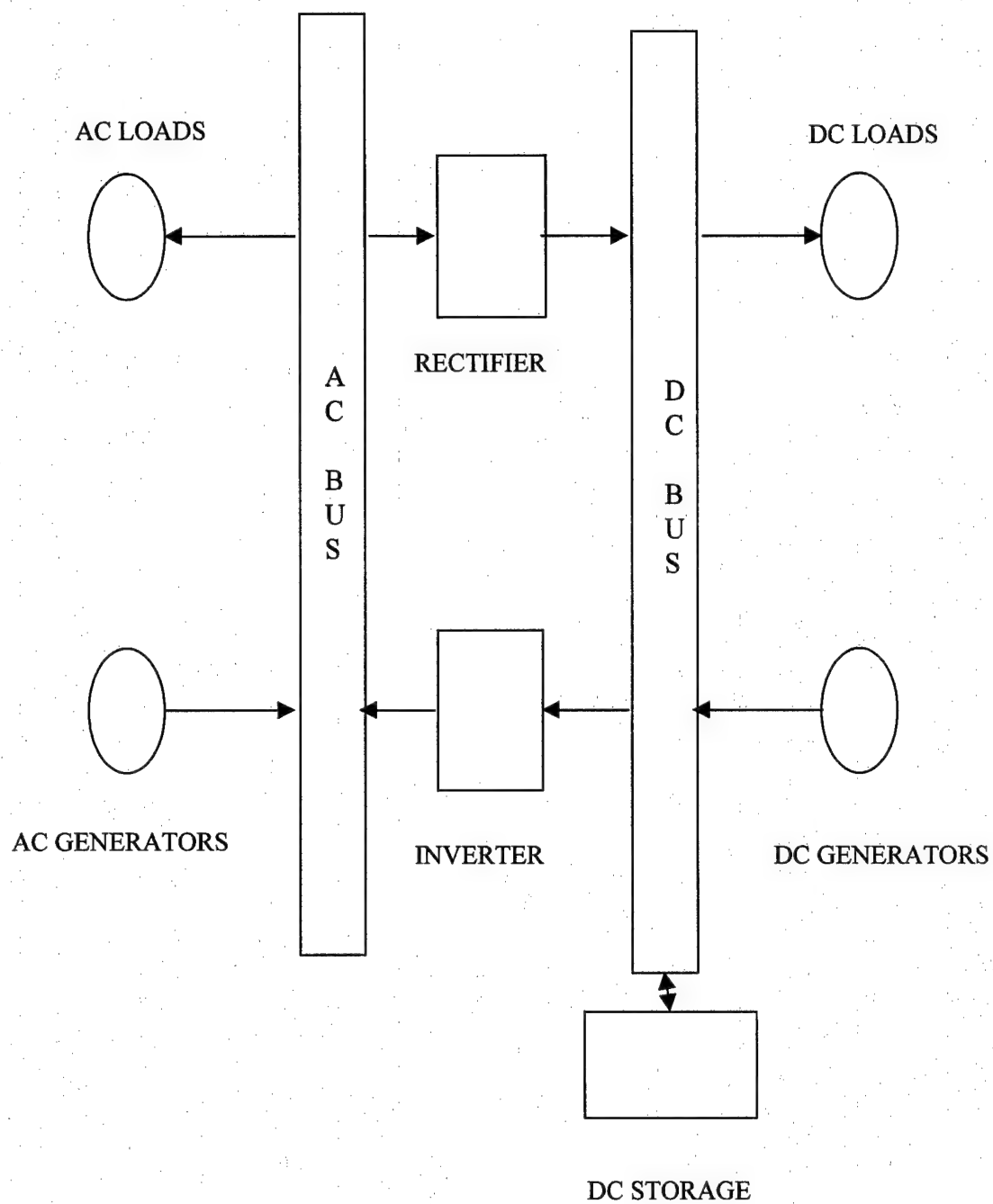


Figure 1. Generic Hybrid Renewable Power System

Loads," represent special fixtures or appliances such as lights, refrigerators, or water pumps that require direct current. Direct current appliances are not common in the United States but are often used with renewable energy systems. Almost all HRPS have an inverter, a rectifier, or both. The inverter, through a special configuration of diodes and other electronic components, converts direct current into alternating current for appropriate end-use by the consumer. The rectifier does the opposite and converts alternating current into direct current for use by DC loads or for storage in the "DC storage" component, namely batteries.

Control of the energy flows into and among the components of this system can be very complicated. For example, in any typical hour, one would have to check the level of the AC and DC loads (electrical demand). If the power provided by AC generators (wind turbines) combined with the inverted DC energy stored in the batteries and the inverted DC generator power is high enough, then the diesel generator could be left off. If the AC and DC generator power is higher than the loads, then one might want to store some of this energy for future use. This would require the

AC energy be rectified and stored in the batteries, while the DC energy could be stored directly. If the AC and DC loads happen to exceed AC and DC generation and battery state-of-charge, then the diesel engine must be turned on to meet the net load. In this case the challenge is to determine the power level the generator should be set at in order to meet, most economically, the net load, taking into consideration future net loads.

### 1.3 Problem Statement

One barrier to using renewable technologies in rural villages is the lack of analytical tools that accurately compare both conventional and renewable energy supply options. Assessing the least-cost mix of supply technologies is a difficult analytical problem that depends on several factors. Among them are the quality and uncertainty of the various wind and sun resources, the costs of acquiring equipment, fuel costs, and site specific descriptions of the daily and seasonal variations in the electrical demand. Consequently, inexperienced users who want to investigate the technical and financial performance of hybrid renewable power systems have a difficult time

choosing the right one for their needs. To aid in choosing a HRPS, screening and analysis models, such as HYBRID 2 (Barley 1996, 66) and the Hybrid Optimization Model for Electric Renewables (Lilienthal 1995), are available. They are useful for feasibility studies that provide data on several topics including comparison among existing technologies in terms of economics and performance leading to an optimal design for any particular application. HYBRID 2 is a detailed simulation model which analyzes system performance over time, once an architecture and dispatch strategy have been decided upon. HYBRID 2 is very time consuming to set up and run, so it is important to screen out sub-optimal system designs avoiding unnecessary simulation effort. One such screening model is the Hybrid Optimization Model for Electric Renewables (HOMER) developed by Peter Lilienthal of NREL. HOMER determines a set of optimal or near optimal system designs, including appropriate dispatch strategies, for further analysis by HYBRID 2. There has been an ongoing effort to implement a predictive dispatch capability, such as the one presented here, into HOMER. While the effort here is a general framework, HOMER is the platform into which this framework

will be incorporated at some future time. A description of HOMER is provided in Appendix A to give the reader an idea of the similarities and differences between the general framework and HOMER.

Screening models such as HOMER determine the lowest life-cycle-cost (LCC) HRPS through analysis of architecture and dispatch strategies. A sound dispatch strategy, the decision when, and if, to turn the generator on, how long it runs, and whether or not to use batteries, can contribute as much to minimizing LCC as the selection of system components (Lilienthal 1995). Given a net load (village electrical demand minus currently available renewable power), typical dispatch strategies considered are 1) load-following--just meeting the net load; 2) cycle-charging--exceeding the net load in order to charge the batteries, or 3) combined--load-following or cycle-charging based on a predetermined set of rules. Chapter 3, section 7 contains a complete description of these strategies.

The problem with existing dispatch strategies is that dispatch decisions are not optimal in any sense. They are based on heuristic rules that employ minimum/maximum battery state-of-charge thresholds or minimum/maximum generator run-



time criteria to determine generator settings. Furthermore, dispatch decisions are made during the current time period regardless of the expected net load next period. An improved implementation makes a dispatch decision now, taking into account the future net loads and optimizing dispatch. At a minimum dispatch decisions based on mean values of future net loads should help to reduce system costs by taking advantage of a "good" system status to mitigate the effects of a "bad" system status some time in the future. In practice, the future net loads are stochastic and one might only know the mean and variance of the net loads in the future, described by some probabilistic distributions.

#### 1.4 Objectives

The goal of this research is to develop a general framework for determining optimal dispatch strategies for the operation of hybrid renewable power systems. The general framework will serve three purposes. The first purpose is as a tool to be incorporated into screening models for determining optimal dispatch strategies for a specific hybrid renewable power system. The second purpose

is to provide an operator a simple, understandable decision making tool for determining how to run a hybrid renewable power system, given battery state-of-charge, electrical demand, renewable resource, and time-of-day considerations. The third is to provide a baseline for future implementation of the framework into a PC-based controller.

The framework developed here is unique in that it simultaneously considers generator costs, battery wear costs, and loss of load penalties in dispatch decisions where other models consider only some subset of the three. This gives the customer the flexibility to trade service availability for lower operating costs and vice versa, if so desired.

With this model, dispatch decisions can be made independently of both the initial and final conditions of the system. This is significant for two reasons. The first reason is that the system user does not have to make potentially sub-optimal decisions about battery charge levels or generator settings at the beginning or end of a planning period. The second reason is that, in the event of a system stoppage, optimal dispatch decisions can be made immediately, regardless of the state of the system or the

experience of the operator. Other models require specification of initial and terminal system conditions or, in the event of system failure, significant operator involvement in re-establishing optimal dispatch conditions.

This framework is predictive in nature. It accounts for the stochastic nature of the wind and sun resources and electrical demand anticipating potential net load realizations in future periods. In doing so, it considers both the immediate and long-term effects of any dispatch decision. Other HRPS models are deterministic in nature or only consider the impact of net load realizations over a short planning horizon.

The model is also flexible enough to consider any nonlinearities in cost or constraint functions where many other models must use piece-wise linear approximations to nonlinear functions.

## Chapter 2

### REVIEW OF RELEVANT MODELS

#### 2.1 Other HRPS Models

While the economic value of forecasting wind has been established in relation to utilities incorporating wind turbine systems into existing grid structures (Miligan 1995, 10), little effort has been given towards using knowledge of expected net load distributions for predictive dispatch of hybrid renewable power systems.

Due to the difficult nature of solving such stochastic models, many dispatch models are deterministic in nature. In fact, of the hybrid renewable power systems simulation software available on the market, none have a predictive dispatch capability (Barley, 1993).

One deterministic approach to dispatch for hybrid renewable power systems is presented in Lilienthal (1995) where required diesel power in each of 8,760 hours of operation is solved for simultaneously using past, current, and point estimates of future net load values. While illuminating, this approach does not account for the

stochastic nature of net loads. This linear programming approach to dispatch does provide automatic sensitivity analysis of important parameters in the model. It also serves as the framework around which the current version of HOMER was built.

Hancock (1995) develops a model of an autonomous remote area power system (RAPS) consisting of photovoltaic, diesel generator, and battery storage components. His model optimizes dispatch strategy over a 24-hour period using discrete one-hour decision intervals. Hancock considers generator fuel cost using a linear fuel consumption curve, but does not consider battery wear cost. He does consider loss of load (not meeting demand) by applying an incremental loss of load penalty (*llp*):

$$llp = 5 \times (\text{incremental fuel consumption}) \times (\text{cost of fuel}) \text{ \$/kWh}$$

His method determines a dispatch strategy using a mixed linear programming-dynamic programming (LPDP) methodology after Bannister and Kaye (1991) to model the remote solar-diesel-battery system. The method uses a typical dynamic programming approach where linear programming is used in

each time step to determine a "cost-to-go" function. The "cost-to-go" is a function of the storage level (battery charge), which, in turn, determines an appropriate generator setting. The LPDP method is a deterministic approach that assumes perfect knowledge of future net loads. Hancock's model combines all of the familiar features of linear programming and the flexibility of dynamic programming to determine optimal dispatch decisions over the specified planning horizon.

Kandil (1991) models a HRPS consisting of solar, wind and conventional diesel generation, as well as a storage capability. He optimizes the dispatch of the autonomous system over a period of 24 hours using discrete one-hour time intervals. He considers generator cost, but not battery wear cost. Loss of load is addressed by using a system constraint that requires demand must be met. Kandil uses a minimum cost generalized network flow model to minimize:

$$TEC = \sum_i \sum_j OMC \times FL_{ij} + FUELC \quad (2.1)$$

The terms in equation 2.1 are defined in table 1.

Table 1. Explanation of Terms (Kandil)

TERM	MEANING
$TEC$	= Total Energy Cost
$OMC$	= Operation and Maintenance cost
$i$	= $i$ th operating hour
$j$	= $j$ th renewable component (PV panel, wind turbine, etc)
$FL_{ij}$	= Power output of renewable energy source
$FUELC$	= Fuel cost of diesel

In order to use a network flow to model the non-linear functions, he uses a piece-wise linear approximation of non-linear cost function, specifically the diesel fuel cost (FC) curve which is expressed as

$$FC = aP^2 + bP + c$$

where  $P$  is the power setting of the diesel system in any one hour and  $a$ ,  $b$ , and  $c$  are constants. The new TEC then becomes

$$TEC = \sum_i \sum_j OMC \times FL_{ij} + \sum_k P_k C_k$$

where  $k$  is the number of segments in the piece-wise linearization,  $C_k$  is the unit fuel cost for segment  $k$ , and  $P_k$  is the power output in segment  $k$ . The network is deterministic in nature in that wind, sun and demand are assumed to be known with certainty.

The results of his model show that the generator is never turned on in the 24-hour period over which the system is optimized. This is significant in that generator fuel usage is the largest variable cost of running a HRPS. The result is not unexpected, however. The system starts with 1000 kWh of charge in the batteries and the highest demand in any one-hour is 300 kWh. Plus, the wind and solar resources exceed demand in 12 of the 24 hours. So, the chance that the generator would have to be used for meeting excess demand was remote to start with.



Barley (1996) develops a simple, understandable heuristic to address dispatch strategies. He considers a HRPS composed of wind turbines, a diesel generator and battery storage. The objective of his work is to minimize

$$C_{op} = c_f \sum_{i=1}^N (F_i \Delta t) + c_{bw} battcap \sum_{i=1}^N [soc_{i-1} - soc_i]^+ \quad (2.2)$$

The terms in equation 2.2 are defined in table 2. Barley analyzes actual data collected at three HRPS sites to gain insight into the level of improvement that can be gained by having an "ideal" predictive dispatch strategy. In doing so, Barley analyzes data for an entire year and uses the data as a "yardstick" against which to measure his heuristic dispatch methods.

His heuristic uses wind-to-average load ratios (WLR), diesel-to-peak load ratios (DLR) and fuel-to-battery cost ratios (FBCR) to determine an appropriate dispatch strategy, specifically load-following or cycle-charging. The ratios are defined as

$$WLR = \frac{\text{Average Wind Power}}{\text{Average Load (Demand)}}$$

$$DLR = \frac{\text{Diesel Size}}{\text{Peak Load (Demand)}}$$

$$FBCR = \frac{Bc_f}{c_{bw} + Ac_f\left(\frac{1}{\eta_{RT}} - 1\right)} \quad (2.3)$$

In equation 2.3, B is the diesel fuel consumption rate at no load (the "cost" in terms of fuel consumption for turning the generator on) and A is the incremental generator fuel consumption rate.

Barley utilizes a linear diesel fuel consumption curve and assumes the diesel generator is sized to at least meet peak electrical demand. Therefore, the potential for loss of load is minimized to the greatest extent possible under his framework. Barley's is a simple and effective approach to dispatch, but does not formally show optimality. His analysis considers a number of sites and is based on empirical data in contrast to the more general Markov Decision Process framework presented here.

Contaxis and Kobouris (1991) address prediction of wind and electrical demand for a wind/diesel hybrid system currently in use in the Greek Islands. The HRPS they consider has several diesel generators and wind turbines,

Table 2. Explanation of Terms (Barley)

TERM	MEANING
$C_{op}$	= Operating cost for simulation period
$c_f$	= Diesel fuel cost
$F_i$	= Diesel generator fuel consumption rate
$\Delta t$	= Time step, hours
$c_{bw}$	= Cost of battery wear
$battcap$	= Battery Capacity
$soc$	= State of charge of the battery
$i$	= Index of time step
$N$	= Total number of time steps in simulation period
$[\dots]^+$	= Indicates summation of positive term only

but no storage capability (batteries). The system is assumed to be autonomous, requiring no manual dispatch. Unit commitment, or determining the number of generators to utilize over a fixed, finite planning horizon ( $T$ ), is the focus of their work. Their model uses a two step process: the first step is to forecast the electrical demand and wind resource over the planning horizon; and the second step is to use a heuristic algorithm to determine unit commitment in any given time period.

The wind and demand forecasts are assumed to be normally distributed random variables and are forecast using ARMA models. Contaxis and Kobouris found forecasting the wind resource to be a difficult undertaking. Forecasting demand, however, is more reliable since demand can be represented by a stationary process with strong regularities.

Rather than using a heuristic algorithm, Bakirtzis and Gavanidou (1992) use a more formal approach to predictive dispatch. They analyze the operation of a HRPS in the context of a stochastic dynamic program. In essence, they use a very dense decision tree and roll it back to determine optimal dispatch strategies. The system under consideration

has multiple diesel generators and wind turbines as well as PV and battery storage. The goal of their analysis is to determine diesel generator unit commitment and to provide a scheme for optimal operation of the system. Specifically, their objective is to minimize the expected value of fuel

consumption,  $E\{F_T\} = E\left\{\sum_{t=1}^T \left[\sum_{i=1}^{N_D} F_i(P_{D_i}(t))\right]\right\}$ , over a 24-hour period.

The terms of the objective function are explained in table 3. They use a linear fuel consumption curve, but do not consider battery wear costs. Loss of load is a minor issue in their model since committing an additional generator can avoid loss of load.

Bakirtzis and Gavanidou find their prediction of demand to be satisfactory, but find wind and sun resources to be difficult to predict. For defining their state-space they assume that the wind, sun, and demand distributions are normally distributed and statistically independent. This facilitates calculation of the transition probabilities from one state to the next, because the transition probability is simply the product of the three component probabilities.

Table 3. Explanation of Terms (Bakirtzis)

TERM	MEANING
$F_T$	= Fuel cost over period of length $T$
$t$	= Time index
$T$	= Planning horizon
$i$	= $i$ th diesel generator
$N_D$	Total number of generators
$F_i$	= Fuel used by the $i$ th generator
$P_{D_i}(t)$	= Power setting of the $i$ th generator in time period $t$

They use a discretization method for wind, sun and load distributions, similar to the one used here. The result of their modeling effort is a "look-up" table which has optimal battery state of charge settings for each hour of the day with diesel generator settings that are calculated separately.

Their assumption of a specific time horizon has an effect on the initial and final states of their system and the optimal action at time,  $T$ . Specifically, Bakirtzis and

Gavanidou assume that the battery bank must be fully charged to a level of 400 kWh at the beginning and end of the 24-hour period. Such assumptions of where the system should "start" and where it should "end" in terms of system status could possibly lead to sub-optimal solutions. By assuming a specific time period over which the system must be optimized, it may well be that the "optimal" dispatch up to and including period  $T$  may produce sub-optimal conditions for periods  $T+1$  onward. The necessity of making system status assumptions and time horizon specifications can be avoided by assuming the HRPS can be modeled as an infinite time horizon, Markov Decision Process (MDP).

## 2.2 Model Presented Here

Assuming an infinite time horizon MDP is advantageous because its Bellman equations

$$V(s) = \max_{a \in A} [u(s,a) + \beta \int V(s') p(ds'|s,a)] \quad (2.4)$$

have attractive properties. In particular, if we define the right hand side of equation 2.4 as the Bellman operator,

$$\Gamma(W)(s) = \max_{a \in A} [u(s, a) + \beta \int W(s') p(ds' | s, a)]$$

equation 2.4 can now be written as,  $V = \Gamma(V)$ , which means  $V$  is a fixed point of  $\Gamma$ . This implies that for the discrete dynamic system model used here, unique equilibrium states for each state-value  $V(s)$  exist. Therefore, starting from any initial state, the system will evolve towards equilibrium (Rust 1996, 635). A direct result of this is that the policy given by the solution of the infinite horizon MDP constitutes an optimal decision rule for each state the system can occupy (Rust 1996, 636). Thus, unlike heuristic methods that may only be optimal under specific conditions, the algorithm is optimal regardless of the conditions.

The model developed here can accept any combination of linear or non-linear cost or constraint functions and can evaluate complex, non-linear relationships and achieve globally optimal solutions. The distributions of the wind, sun, and demand that make up the net load distribution (energy demanded minus energy provided by renewable resources) can be anything, meaning statistical independence of the three components is not required as long as the net



load distribution can be discretized. Furthermore, the framework anticipates the net load realizations, so it is predictive in nature. The model assumes the system can be run autonomously, but the product of model also allows for simple, manual dispatch of the system. The look-up tables it produces are optimal generator settings that allow an inexperienced operator to run the system.

Generally, the HRPS considered for remote villages have only one generator and therefore do not have the luxury of using one more than one unit, so unit commitment is not considered although it could easily be incorporated into the model.

The framework is flexible in the sense that loss of load cost, the size of which is determined by the magnitude of the loss of load penalty, can be traded against generator and battery wear costs, if desired. Net load profiles are examined where loss of load is likely to occur, giving a realistic set of results to examine.

The disadvantage of using an MDP is that it is computationally expensive relative to a linear programming or a finite dynamic programming approach to the dispatch problem. This is due to the "curse of dimensionality" (Rust

1996, 625) encountered in defining too large a state-space and/or action set. Also, by discretizing the state-space and action set, the optimal solutions contained here are limited to those combinations of state, action and net load realizations defined by the state-space. It can be shown, however, that fairly "coarse" discretizations do a good job of approximating underlying continuous functions (Rust 1996, 659). Figure 2 shows a comparison of this framework to the other models described here.

	Model presented here	Lilienthal	Hancock	Kandil	Barley	Contaxis & Kabouris	Bakirtzis & Gavanidou
Considers Battery Cost	X	X			X		
Considers generator cost	X	X	X	X	X	X	X
Considers loss of load	X		X	X			
Considers manual dispatch	X						
Accepts linear functions	X	X	X	X			X
Accepts non-linear functions	X						X
Dispatch indep.of starting conditions	X						
Dispatch indep.of final conditions	X						
Predictive	X					X	X
Stochastic	X					X	X

Figure 2. Comparison of HRPS Models

## Chapter 3

### MODELING THE HYBRID RENEWABLE POWER SYSTEM

In this chapter I analyze the hybrid renewable power system dispatch problem from a theoretical approach as suggested by Ethridge (1995, 133) and then outline the specific assumptions made in modeling the HRPS. The problem of operating a HRPS can be framed in terms of a sequential decision making process over time. Consequently, it is well suited for treatment as a dynamic program, specifically, a Markov Decision Process.

#### 3.1 Dynamic Programming

Dynamic programming is a useful framework that practitioners apply to a variety of multi-stage decision problems occurring over time or when choices are made sequentially. The operation of a HRPS fits nicely into this category. Problems that are formulated and solved using dynamic programming have several common characteristics. They are shown in table 4.

Table 4. Typical Characteristics of a Dynamic Program

Characteristic	Notation
Time Index (or Stages)	$t \in \{0,1,2,\dots,T\}, T \leq \infty$
State-Space	<p><math>S</math> such that <math>s \in S, s' \in S</math></p> <p><math>s</math> is a state the system is currently occupying</p> <p><math>s'</math> is a state to which the system can transition</p>
Action Set	<p><math>A</math> such that <math>a \in A</math></p> <p><math>a</math> is an action available to the decision-maker in state <math>s</math></p>
Cost	$C_{ss'}^a$ the cost of going from $s$ to $s'$ upon taking action, $a$

The status of the system is described by a state-vector containing variables that adequately describe the evolution of the system up to a particular point in time. If the future is known with certainty, i.e., it is deterministic, or for that matter even if the future is uncertain, an optimal dispatch strategy or policy, prescribing what action to take from the current state, can be determined.

In the deterministic case, any action taken determines the next state of the system. In the case where the outcome of a decision is uncertain and a probability distribution

for occupying a state at the next stage is known, the system is considered to be stochastic (White 1969, 24). In the stochastic case, probability transition matrices are introduced into the dynamic programming framework. They describe the chances of the system transitioning from  $s$  to  $s'$  given some action taken in state  $s$ .

With a stochastic dynamic program, the actual decision path now depends on how the random aspects of the system manifest themselves. Because of this, "solving" a stochastic dynamic program involves giving a decision rule for every possible state of the HRPS, not just along a single optimal path through time which is the result of solving the deterministic case (Trick 1998).

For finite time dynamic programs ( $T < \infty$ ) finding the optimal solution is accomplished by evaluating the operation of the system, one stage at a time, starting with the last stage and working backwards in time. For infinite time horizon dynamic programs ( $T = \infty$ ) other solution methods are used to find the optimal solution. Some of the methods available for solving infinite time MDPs are successive approximations, accelerated successive approximations using error bounds, and various forms of Policy Iteration (Rust

1997, 653-658). Regardless of the time scale of the dynamic program, the optimal policy ( $\pi^*$ ) obtained is one that determines at each stage for any state, a unique action to be taken from among the allowable actions (White 1969, 29). The policy is determined by using Bellman's principle of optimality (Bellman 1957). Succinctly stated by Lapin (1985) as: "The optimal policy must be one such that, regardless of how a particular state is reached, all later choices proceeding from that state must be optimal."

### 3.2 Markov Decision Processes

The problem considered here is suitably framed as a finite, discrete, infinite time horizon stochastic dynamic program, commonly referred to as a Markov Decision Process. Such a dynamic program, in addition to the characteristics described in table 4, has a set of transition probabilities that determine the movement of the system through its state-space.

The MDP has a "finite" number of states,  $S = \{1, 2, 3, \dots, N\}$ , with transition probabilities,  $P_{ss'}^a = p(s'|s, a)$ , from some initial state,  $s \in S$ , to another state,  $s' \in S$ , being dependent only on the initial state, and an action,  $a$ ,

taken from a set of  $n$  finite actions,  $A = \{a_1, a_2, a_3, \dots, a_n\}$ , available to a decision-maker in state  $s$ . For this framework, the number of actions in any state must be finite, but the number of actions available in each state may be different from one state to the next (Howard 1960, 28).

The state descriptions are an especially important aspect of framing a problem as a Markov Decision Process. The state should inform us of as much about the environment as possible in a way that summarizes the relevant history of the process completely and compactly. In particular, if certain events that have a bearing on the decision to be made have happened prior to a specific stage in the decision process, then the state description must record these events (White 1969, 32). A state description that succeeds in retaining all relevant information is said to be Markovian (Sutton 1998, 61-62). This means the probability of moving from one state to another state is independent of time. This "memoryless" property can be assured with proper state descriptions using a state-vector that captures the salient features of the system transitions up to the point in time where an action is taken.



If the time between transitions is the random variable of interest, the system could be considered a continuous time process (Howard 1960, 93). Then, the significant parameters of the HRPS process would be transition rates,  $b_{ij}$ , of going from state  $i$  to state  $j$  in the time interval  $dt$ , resulting in a transition probability of  $b_{ij}dt$ , ( $i \neq j$ ). For this model, however, the interest is in the state transitions due to net load realizations.

Since the focus of this work is on state transitions based on net load realizations, it is convenient to assume that the time between transitions is a constant and we therefore index state transitions in time (Howard 1960, 3). Thus, the "discrete" terminology applies to the stage structure; i.e., decisions are made at discrete time intervals.

Analyzing a decision in terms of a Markov process is more valid in some applications than others. In applications where outside forces are minimal and where policies are expected to operate under stable conditions for a long time, an analysis based on a Markov process is appropriate.

Stable conditions are assumed for this analysis of hybrid renewable power systems. By doing so, it is assumed the wind and sun resource profiles of a particular site remain stable indefinitely, barring any permanent meteorological changes at the HRPS site. In addition, given the relatively simple lifestyle of people in remote villages in developing countries, the electrical demand profiles are assumed to remain relatively stable over time. There is a caveat to this last statement, however. In my opinion, as villagers become used to availability of electrical power, their demands for the power may increase. Any significant changes in electrical demand profiles would require re-running the model to obtain a new policy.

I assume the policies for energy dispatch of a HRPS under a Markov Decision Process framework face stationary conditions, i.e. the transition probabilities are time invariant and only dependant on the state of the system when a decision is made. Hadley (1964, 454) confirms that if the distribution of the random variables do not change from period to period, then it is possible to use an infinite planning horizon and the influence of decisions at all future times are felt in the current decision. More weight

can be added to this infinite planning horizon assumption by considering that each of 8,760 hours in a year over a typical system life of approximately 20 years is evaluated in the sequential decision process. This amounts to 175,200 time periods or stages, which in my opinion is quite large by any planning horizon standards.

If an environment has the Markov property (Sutton 1998, 63), it follows equation 3.1,

$$P_{ss'}^a = \Pr\{s_{t+1} = s', r_{t+1} = r | s_t, a_t\} \quad (3.1)$$

where  $P_{ss'}^a$  is the probability of going from  $s$  to  $s'$ , under action,  $a$ . Its one-step dynamics then enable us to predict the next state and expected next cost,  $\sum_{s'} P_{ss'}^a C_{ss'}^a$ , where  $C_{ss'}^a$  is the cost of going from an initial state to a successor state under some action. The Markovian property makes it is possible to predict all future states and expected costs from knowledge of only the current state, as if we had a complete history of the system. This implies the best policy for choosing actions as a function of a Markov state is just as good as the best policy for choosing actions

knowing the complete history of the system (Sutton 1998, 63).

### 3.3 Model Specifics

This section describes the specific assumptions made regarding the stages, state-space, action set, cost structure, and transition probabilities in the Markov Decision Process formulation of this model.

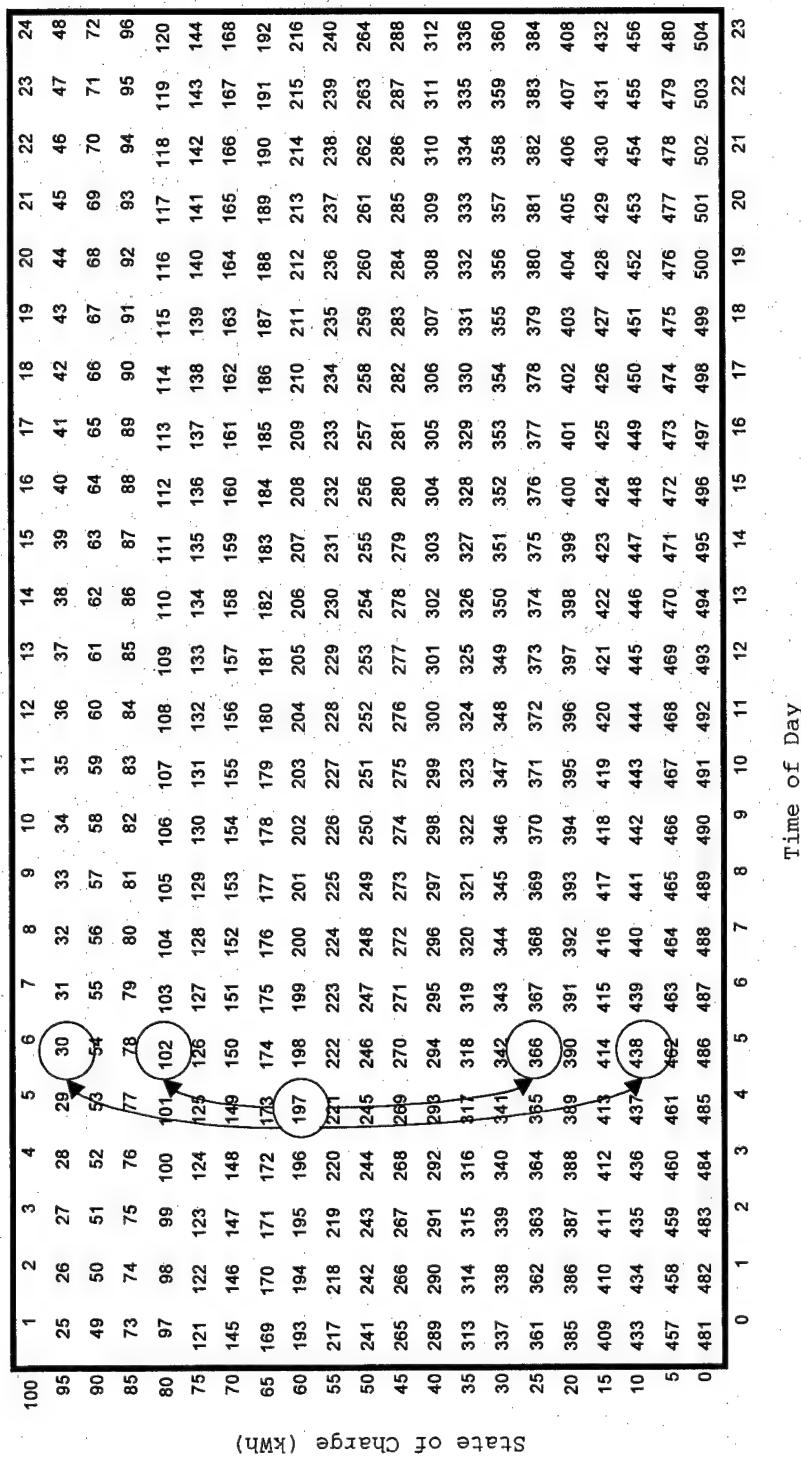
#### 3.3.1 Stages

I assume an infinite time horizon with time divided into discrete one-hour periods or stages. This is a typical time interval used for a majority of modeling work with HRPS (Barley 1996; Lilienthal 1995; Hancock 1994). A one-hour time interval is appealing for two reasons. First, the state-of-charge is measured in kilowatt-hours (kWh). Therefore, in calculating the number of kilowatts removed or added to batteries and an appropriate generator setting measured in kW, a natural time unit to employ is one hour. The second reason is dispatch decisions for generators should reflect some common sense in terms of turning the generator on and off. For time periods less than one hour,

the constant starting and stopping of the generator, if so prescribed by a policy, could have an adverse affect on engine lifetime.

### 3.3.2 State-Space

For the HRPS, each state in the state space  $S$  is defined with the state-vector {state-of-charge, time-of-day}. Figure 3 shows the state-space with discrete states-of-charge (y-axis) ranging from 0 kWh to 100 kWh. The state-of-charge discretization step,  $\Delta soc$ , is 5 kWh, defining 21 possible states-of-charge: 0, 5, 10, . . . , 100 kWh. The time-of-day ranges from 0 (midnight) to 23 (11:00 pm), covering a 24-hour period. Consequently, there are  $21 \times 24 = 504$  states. Certain advantages lie in defining a state-space of this size. The most important of which is that for relatively small scale problems, where  $S \approx 500$ , and the discount factor,  $\beta$ , is sufficiently close to 1 (i.e.  $\beta > .95$ ), the Policy Iteration algorithm is regarded as one of the fastest methods for computing the state-values and finding an optimal policy (Rust 1996, 654). The state-space



is "fine" enough to give accurate and insightful results without creating computational difficulties. Even with a somewhat coarse discretization of the state-space, a good approximation of the underlying function can be achieved (Rust 1996, 659).

Also, using state-of-charge versus time-of-day is intuitively appealing. When considering transitions from one state to the next, the state-space definition only allows transitions from one column to the next in a spatial sense (left to right in figure 3). With net load as the stochastic element in the model, state-of-charge is affected in a vertical sense (up or down), as actions and net load realizations charge or discharge the batteries. For example, from state 197 in figure 3, the only transitions available are to states 6, 30, 54, 78, 102, 126, 150, 174, ..., 486. This is true because the time-of-day transitions from hour 4 to hour 5 (left to right) with a probability of 1. Therefore, the set of states to which the system can transition is restricted to the column that represents, time-of-day = 5. In state 197, the state-of-charge at the end of hour 4,  $soc_4$ , is 60 kWh. Depending on the action taken,  $a_4$ , at the end of hour 4 and the net load

realization,  $NL_5$ , over the next hour, the state-of-charge at the end of hour 5,  $soc_5$ , is either higher, lower, or the same.

$$soc_5 = soc_4 + a_4 - NL_5$$

if  $a_4 \geq NL_5$  then  $soc_5 \geq soc_4$

else if  $a_4 \leq NL_5$  then  $soc_5 \leq soc_4$

Another advantage of defining the state-space this way lies in the computational efficiency that can be gained by such a definition. The Policy Iteration algorithm (see chapter 3, section 5) requires a complete sweep of the entire state-space calculating expected values for each state. This equates to making calculations for each of the 504 states for each policy evaluation and policy improvement step. With the state-space defined here, calculations need only be made for 21 of 504 or 4% of the states in each sweep. This significantly reduces computation time, as 96% of the states in the state-space do not have to be evaluated in each iteration.

Furthermore, defining the state-space with the variables state-of-charge and time-of-day results in a



simple table, the interior of which contains the optimal action that an operator should take given any combination of the two state variables. The operator simply checks the time-of-day and a gauge measuring the state-of-charge of the batteries, and uses the table to determine the appropriate generator setting. Such a process avoids even the simplest mathematical calculations that might be involved with the use of a state-space that incorporates additional state variables.

This is an important aspect of this model. While relatively parsimonious from a modeling standpoint, the simplicity cannot be overstated. In the event that manual dispatch of a HRPS is needed, such a table provides a simple, and in my opinion, understandable alternative to automated dispatch. With little or no technical training, even the most inexperienced person could operate the system manually.

### 3.3.3 Action Set

There are essentially two actions available to an automatic controller or a system operator: to leave the generator off or turn the generator on. If the generator is

turned on, the decision as to what power level to set it at must be considered. For this model I assume discrete generator levels ranging from 0 kW to 100 kW. The generator discretization step,  $\Delta a$ , is equal to 5kW, matching the discretization step for the state-of-charge,  $\Delta soc$ . Consequently, there are 21 actions 0, 5, 10, . . . , 100 kW available in each state. By setting  $\Delta a = \Delta soc = \Delta NL$  (the net load discretization step), this ensures that any combination of initial state-of-charge, action, and net load realization puts the system in one of the states defined previously.

### 3.3.4 Cost Structure

The cost structure in this model is composed of costs,  $C_{ss'}^a$ , incurred as a result of taking a specific action and transitioning from  $s$  to  $s'$ . There are three types of costs that can be incurred: diesel generation costs, battery wear costs, and loss of load costs.

Diesel generation costs are incurred any time the generator is turned on. They are calculated using a linear diesel fuel consumption function (equation 3.2) defined by Skarstein and Uhlen (1989, 72-87). Skarstein and Uhlen looked at several models of diesel generators and developed

a linear relationship between the rated power of each diesel,  $p_r$ , and its fuel consumption characteristics.

$$F = .08415p_r + .246p \text{ liters/kWh} \quad (3.2)$$

Here  $p_r$  is the maximum generator power, fixed at 100 kW for this model, and  $p$  is the power output in any particular hour. The fuel consumption function is multiplied by a fuel cost ( $c_f$ ), say \$.264/liter, to arrive at a fuel cost per kilowatt hour:

$$\begin{aligned} \text{FuelCost} &= c_f F = .264[.08415(100) + .246p] \\ &= 2.22 + .0649p \end{aligned} \quad (3.3)$$

In addition to the fuel cost, an operation and maintenance (O&M) cost must be considered when the generator is turned on. I assume this cost is \$1 per hour of operation (Lilienthal, 1998). For example, the cost of running the generator at 50kW for one hour would be

$$\text{FuelCost} = 2.22 + .0649(50) = \$5.465$$

$$\text{O\&M} = \$1$$

$$TotalGeneratorCost = 5.465 + 1 = \$6.465 \quad (3.4)$$

Battery wear costs ( $c_{bw}$ ) are incurred when the net load is positive (meaning electrical demand is greater than the available renewable resources) and the generator is not turned on. Battery degradation is actually a non-linear function of the state-of-charge at the time the battery is charged or discharged (Barley 1993). For this model I assume that the battery wear costs are a linear function of the change in the state-of-charge and are applied when the battery is discharged. This is a typical assumption used in modeling HRPS (Lilienthal 1995; Barley 1996).

The "loss of load" cost is incurred when the action taken in combination with the state-of-charge is not enough to satisfy a positive net load. This leads to unmet electrical demand. The loss of load cost or penalty serves two purposes. Most importantly, in any optimization model where minimization is the objective and a lower bound is not specified, the decision variables will be set to zero. This results in a trivial solution to the problem.

The loss of load cost or penalty can also be thought of as a measure to ensure some level of service to the

consumer. It ensures that electricity is provided to the greatest extent possible, depending on the magnitude of the penalty and the stochastic nature of the net loads. For developing countries, and specifically small villages, the cost of unmet electrical demand is difficult to quantify, so sensitivity analysis is performed to determine an effective range of penalties. Preliminary model verification with a deterministic net load profile indicated that a loss of load penalty on the order of \$.30 is sufficient to avoid loss of load altogether. Model verification also indicated that penalties greater than \$.75 did not change the optimal strategy, all other parameters being equal. For the stochastic model, I broaden the range and analyze the effect of different costs per kilowatt-hour ranging from \$.25 to \$1.

### 3.3.5 Transition Probabilities

The distribution of net loads ( $NL$ ) that could be realized after an action is taken determine the transition probabilities of going from one state to the next. The stochastic nature of the net loads is a result of the combination of electrical demand, available wind resource,

and available sun resource. After an action,  $a_t$ , is taken, the realization of the net load,  $NL_{t+1}$ , interacts with the state-of-charge,  $soc_t$ , to either charge or discharge the batteries.

For this model, I assume that the net load profiles for each hour of the day are exogenous to the model. My assumption is that the profiles are provided by the customer and contain site specific data for a potential HRPS site. Contributions to the net load profiles by sun and wind resources can be gathered directly or estimated using sun and wind resource maps. The electrical demand contribution can be estimated for potential sites using villages of similar size, or for sites where diesel generators already exist, electrical demand can be measured directly. The three components of net load combine to produce a net load distribution. My framework accepts net load distributions with any combination of scale or location parameters as long as they can be appropriately discretized. Here, I assume each profile is normally distributed,  $NL \sim N(\mu, \sigma^2)$ , with a prescribed mean ( $\mu$ ) and variance ( $\sigma^2$ ). I consider net loads in the range:

$$\mu_{NL} - 6\sigma_{NL} \leq NL \leq \mu_{NL} + 6\sigma_{NL}$$

for each hour of the day. Such an assumption captures virtually 100% of the possible realizations of the net load. I discretize each distribution using a method similar to that used by Rust (1996, 659). I calculate probability masses by taking the cumulative probability at a point one-half the net load discretization step,  $\frac{1}{2}\Delta NL$ , above the net load minus the cumulative probability at a point one-half the net load discretization step below the net load. Two exceptions occur at net loads closest to  $\mu_{NL} \pm 6\sigma_{NL}$ . For  $\mu_{NL} - 6\sigma_{NL}$  the probability mass is one-half the net load discretization step above the net load and includes the tail to the left of the net load. For  $\mu_{NL} + 6\sigma_{NL}$  the probability mass is one-half the net load discretization step below the net load and includes the tail to the right of the net load. This approach ensures the realization of each net load is approximately centered on its probability mass and the sum of the probabilities over the possible net loads sum to 1.

To model physical limitations of the batteries, in cases where the state-of-charge, action, and net load would

put the system in states that are higher than battery capacity or lower than zero charge, I accumulate probability at the state closest to the point where the over or under charge occurs. For example, if the state-of-charge is 100 kWh (maximum state-of-charge for this model), the action taken is  $a=100$  kW, and  $NL \sim N(20, 4^2)$  with  $\Delta NL = 5$  kW, the highest possible net load under the assumptions I make is 40 kW. Thus, the starting state-of-charge plus the action taken minus the highest possible net load of 40 kW over the hour, results in a theoretical state-of-charge of 160 kWh.

$$100kWh + 100kW \times 1hr - 40kW \times 1hr = 160kWh$$

This is clearly greater than the highest allowable state-of charge (100 kWh) for this model, so the probability of being in a state-of-charge of 100 kWh is

$$\sum_{i=0}^{40} p_i = p_{soc=100} = 1 \quad \text{for} \quad i = 0, 5, \dots, 40 \quad (3.5)$$

where  $p_i$  is the probability that the net load is  $i$ . For this example, any realization of the net load results in a



state-of-charge of 100kW. Similar calculations occur at the lower boundary of the defined state-space.

### 3.4 Solution of the Model using Policy Iteration

Policy Iteration is an algorithm developed by Howard (1960, 42) that converges on a optimal action for each state as the number of stages becomes very large. Consequently, it may be applied only to continuing processes or to those whose termination is remote. Policy Iteration has simplicity of form and interpretation that makes it very desirable from a computational point of view and often converges in surprisingly few iterations (Sutton 1998, 97). Policy Iteration has the following properties (Howard 1960, 39):

- The solution of the sequential decision process is reduced to solving sets of linear simultaneous equations for the "value" of each state, where value is a measure of all future costs to be expected.
- Each succeeding policy found in the Policy Iteration cycle finds a higher value for a state than the previous policy.

- The iteration cycle will terminate on the policy that has the largest value attainable within the scope of the problem.

Policy Iteration (equation 3.6) is composed of two parts, a policy evaluation operation ( $\xrightarrow{E}$ ) and a policy improvement routine ( $\xrightarrow{I}$ ). The policy evaluation operation yields state-values ( $V^\pi$ ) as function of policy ( $\pi$ ) by solving the Bellman Equations,  $V^\pi(s) = \sum_{s'} P_{ss'}^a [C_{ss'}^a + \beta V^\pi(s')]$ , whereas the policy improvement routine yields the policy as a function of the state-values. Policy Iteration converges to an optimal policy ( $\pi^*$ ) and optimal value function ( $V^*$ ) in a finite number of iterations.

$$\pi_0 \xrightarrow{E} V^{\pi_0} \xrightarrow{I} \pi_1 \xrightarrow{E} V^{\pi_1} \xrightarrow{I} \pi_2 \xrightarrow{E} \dots \xrightarrow{I} \pi^* \xrightarrow{E} V^* \quad (3.6)$$

#### 3.4.1 Policy Evaluation

Policy evaluation, accomplished by solving a system of  $S$  equations in  $S$  unknowns, computes the state-value function  $V^\pi(s)$  for each state under an arbitrary policy,  $\pi$ . The value functions derived from policy evaluation estimate

how good it is to be in a given state. The notion of "how good" is defined in terms of future costs that can be expected. The future costs depend on what actions are taken so, the value functions (equation 3.7) are defined with respect to particular policies where a policy represents the mapping,  $\pi: S \rightarrow A$ .

$$V^{\pi}(s) = \sum_{s'} P_{ss'}^a [C_{ss'}^a + \beta V^{\pi}(s')] \quad (3.7)$$

Equation 3.7, the Bellman equation for a value function  $V^{\pi}(s)$ , expresses a relationship between the value of a state and the values of its successor states. There are 504 Bellman equations for this model, one for each state. Equation 3.7 states that the value of the initial state  $V^{\pi}(s)$  must equal the discounted value of sum over the expected next state-values,  $V^{\pi}(s')$ , plus the expected cost  $\sum_{s'} P_{ss'}^a C_{ss'}^a$  along the way. In an economic sense, the difference in relative values between two states is equal to the amount that a rational person would be willing to pay in order to start his transitions from one state as opposed to another.

These relative values hold the key to finding a better and ultimately the best policy (Howard 1960, 34-36).

### 3.4.2 Policy Improvement

For Markov Decision Processes, we can precisely define an optimal policy using policy evaluation under different policies. Value functions define an ordering over policies, consequently, there is always at least one policy that is better than or equal to all other policies. This is the optimal policy,  $\pi^*$ . It has the state-value function, called the optimal state-value function,  $V^*$ , defined as  $V^* = \max_{\pi} V^{\pi}(s)$  for all  $s \in S$ . This leads to the Bellman optimality equation

$$V^*(s) = \max_a \sum_{s'} P_{ss'}^a [C_{ss'}^a + \beta V^*(s')] \quad (3.8)$$

Simply stated, any policy that is "greedy" with respect to the optimal value function is an optimal policy. The term "greedy" describes policies that select actions based only on their short-term consequences. The nice thing about the Bellman Optimality equations is that if they are used to

evaluate the short-term consequences of actions, then a greedy policy is actually optimal in the long-term sense. This is true because  $V^*$  takes into account the cost consequences of all possible future behavior. By means of  $V^*$ , the optimal expected long-term value is turned into a quantity that is locally and immediately available for each state. Hence the one-step ahead search yields the long-term optimal actions (Sutton 1998, 75-77), so in theory, solving the Bellman optimality equation provides the route to finding an optimal policy (Sutton 1998, 79).

### 3.5 Policy Iteration Algorithm

Theoretically, the simultaneous solution of the Bellman optimality equations provides the long-term values for each state in the state space. In practice this requires solving up to 504 equations in 504 unknowns for this model. To accomplish this task an algorithm is used that allows for the solution of a system of linear equations iteratively (figure 4). This in-place algorithm is a Gauss-Siedel-style algorithm (Sutton 1998, 110) for solving such a system of equations. Initially, an arbitrary value and an arbitrary policy are assigned to each state. Under the policy

1. Initialization

$V(s) \in \mathcal{R}$  and  $\pi(s) \in \mathcal{A}(s)$  arbitrarily for all  $s \in \mathcal{S}$

2. Policy Evaluation

$\Delta \leftarrow 0$

For each  $s \in \mathcal{S}$ :

$v \leftarrow V(s)$

$$V(s) \leftarrow \sum_{s'} P_{ss'}^{\pi(s)} [C_{ss'}^{\pi(s)} + \beta V^{\pi(s)}(s')]$$

$\Delta \leftarrow \max(\Delta, |v - V(s)|)$

until  $\Delta < \epsilon$  (a small positive number)

3. Policy Improvement

"polycystable"  $\leftarrow$  true

For each  $s \in \mathcal{S}$ :

$b \leftarrow \pi(s)$

$$\pi(s) \leftarrow \arg \max_a \sum_{s'} P_{ss'}^a [C_{ss'}^a + \beta V^*(s')]$$

If  $b \neq \pi(s)$ , then "polycystable"  $\leftarrow$  false

If "polycystable", then stop; else go to step 2

Figure 4. Policy Iteration Algorithm (using iterative policy evaluation) for  $V^*$ . Source: Sutton, page 98

evaluation operation the algorithm stores the initial values of each state and then sweeps through the entire state set calculating the long-term value of each state under a specific policy. It does so until the absolute value of the maximum difference between the initial value and the calculated value of each state is less than  $\epsilon$ .  $\epsilon$  is a user defined tolerance that represents the maximum difference that the user is willing to accept between the actual state-value and an approximation of the state-value.

Under the policy improvement routine the algorithm stores the current action for each state (i.e., the policy,  $\pi:S \rightarrow A$ ) and then sweeps through the entire state set calculating the long-term expected value of each state over the entire action set. It then picks the action that gives the highest value to each state. This new action is compared to the initial action. If there is no difference between the two state-values, then this is the optimal policy for that state.

$\beta$  is a factor used to weight future value relative to the present. Its primary purpose, void of any other meaning, is to ensure that state-values converge to a finite value. In fact, as long as  $\beta < 1$  the existence and

uniqueness of the state-value functions  $V^*$  is guaranteed (Sutton 1998, 90). If  $\beta$  is greater than or equal to 1 then the sum of present and future costs would grow without bound.

A common interpretation of  $\beta$  is as a discount factor (Winston 1994; Howard 1960; Rust 1996). If a person values the future a great deal he or she assigns a discount factor close to 1. If a person is only concerned with the present, a discount factor of 0 is appropriate.  $\beta$  for a single period transition is calculated using the following formula

$$\beta = \frac{1}{1+r}$$

where  $r$  is a discount rate. An appropriate discount rate is extremely difficult to determine a priori, especially on projects such as hybrid renewable power systems.

While policy iteration can only be applied to continuing processes whose termination is remote, there is still some possibility that the process might stop. In cases where there is uncertainty concerning the duration of the process,  $\beta$  is considered to be the probability that the



process will continue to incur costs after the next transition (Howard 1960, 77). This implies  $1-\beta$  is the probability that the process will stop at its present stage. This is the interpretation used here.

For this model I use  $\beta=.999$ . This is a value of  $\beta$  that provides a reasonable measure in terms of system reliability, while at the same time keeping model run times to a reasonable length. With  $\beta=.999$  average run times for each scenario are around 4 hrs on a Pentium 166 MHz, IBM compatible platform using Visual Basic for Applications. For  $\beta=.9999$ , one scenario solved in approximately 17 hours while another scenario was not done solving after 30 hours. Another reason  $\beta=.999$  is an adequate choice is that as  $\beta$  approaches 1, which Rust (1996, 631) defines as any  $\beta>.95$ , the state-values converge to those which would be calculated under the long-run average cost MDP formulation:

$$\max E[\lim_{n \rightarrow \infty} (\frac{\text{cost incurred during periods } 1, 2, 3, \dots, n}{n})]$$

This implies  $\beta=.999$  gives results that are, for all intents and purposes, equal to those that would be found under a

solution method that does not account for some probability of system failure.

### 3.6 Solution Procedures

I use the Policy Iteration algorithm to determine the optimal policies ( $\pi^*$ ) for dispatching HRPS for two different net load profiles under nine different scenarios each. Comparisons are made across different fuel costs, battery wear costs, and loss of load penalties.

The first net load profile (figure 5) has positive net

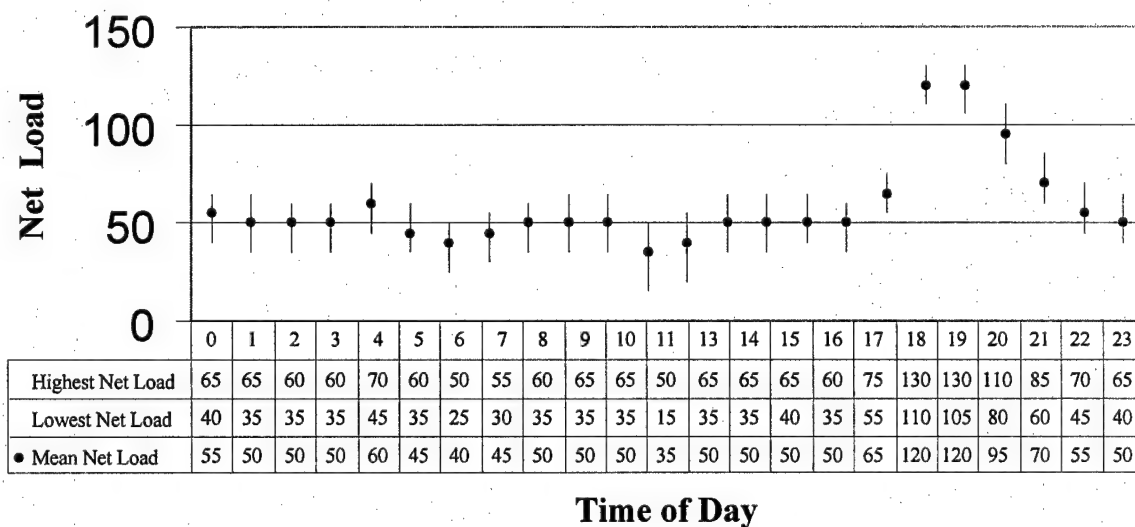


Figure 5. Net Load Profile 1

load values for each of the 24 hours of the day. In hours 18 and 19, all possible realizations of the net load exceed the maximum generator rating (100 kW). In hour 20, some realizations of the net load distribution exceed the maximum generator rating. Both of these situations may potentially lead to a loss of load.

In addition, most of the potential net load realizations exceed the critical load,  $L_c$ , in each of the 9 scenarios (see chapter 3, section 7 for a complete description of the meaning of  $L_c$ ). This means the results of the combined dispatch strategy are the same as the load-following strategy. In general, the individual net load distributions do not show wide variation around their means.

The second net load profile (figure 6) is characterized by some occurrences of mean net loads that are less than zero, meaning the batteries in the system should be allowed to charge, free of generator power, in those instances. There are no instances where any one net load realization exceeds the maximum power rating of the generator, so loss of load should be minimized all other things being equal. In addition, the variances around the individual means are quite large in some instances and quite small in others.

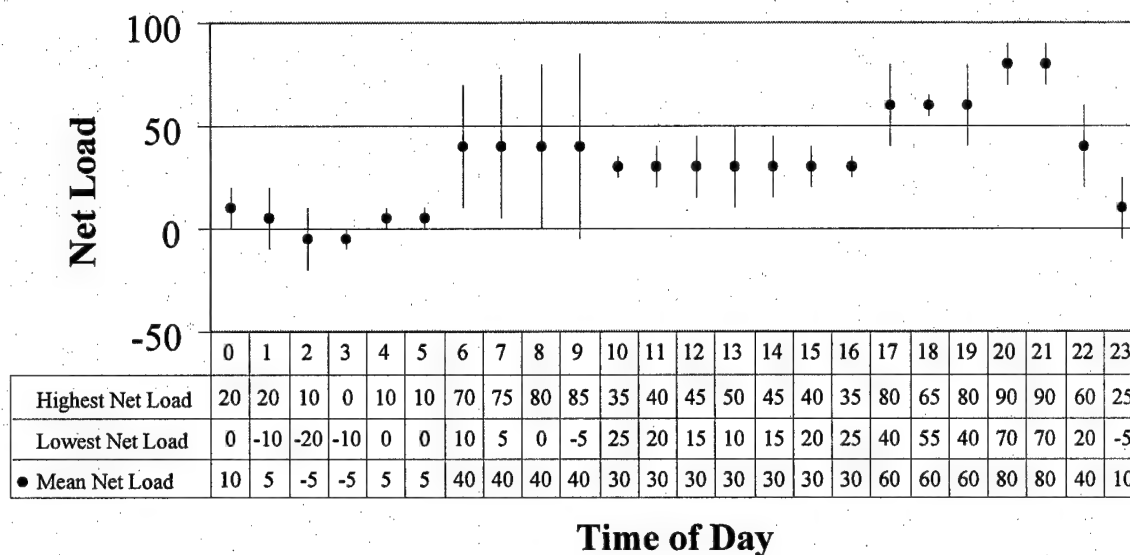


Figure 6. Net Load Profile 2

The potential net load realizations vary around the critical load,  $L_c$ , in each of the 9 scenarios, so the combined dispatch strategy may or may not take the form of one of the other standard strategies.

The various scenarios over which the net load profiles are examined and the resulting optimal policies developed are shown in table 5. The base case and ranges of parameter values with the exception of the loss of load penalty are taken from Barley (1996, 53). A loss of load penalty of \$1 is assigned as the base case parameter.

Table 5. Scenarios

Scenario Number	Battery Wear Cost (\$/kWh)	Fuel Cost (\$/liter)	Loss Penalty (\$/kWh)	Analysis of:
1	.10	.264	1	Base Case
2	.10	.264	0	Loss
3	.10	.264	.25	Loss
4	.10	.264	.50	Loss
5	.05	.264	1	Battery Wear
6	.20	.264	1	Battery Wear
7	.10	.10	1	Fuel Cost
8	.10	.20	1	Fuel Cost
9	.10	.50	1	Fuel Cost

The different scenarios represent variation over different parameters. Runs 1 through 4 vary the loss of load penalty. Runs 1, 5 and 6 vary the battery wear cost ( $c_{bw}$ ), and runs 1 and 7 through 9 vary the fuel cost parameter ( $c_f$ ).

### 3.7 Evaluation of Optimal Strategies

After an optimal policy is developed for a particular scenario, I test the optimal policy versus the load-following, cycle-charging, and combined strategies as well as a random strategy. This is done using a simulation to generate realizations of the net load profiles for each hour of the day. These realizations interact with the state-of-charge of the batteries and the action taken according to the specific dispatch strategy under consideration. The cost structure used in the simulation is identical to the one used in development of this framework, so a comparison of the optimal dispatch strategies to the other dispatch strategies is consistent with the assumptions used in the MDP.

For the optimal strategy, the state-of-charge and time-of-day combination is referenced to the optimal dispatch table and the prescribed action is taken.

Under a load-following strategy, if the batteries cannot meet the net load, diesel power is used to just meet the net load and the batteries are never charged by the diesel. Thus, the state-of-charge does not change in a period where the generator is turned on.

Under a cycle-charging strategy, if the batteries cannot meet the load, the diesel is run at maximum power, where generators are most efficient, to both meet the load and charge the batteries. The intent of this strategy is to run the generator at full power and to top off the batteries with any power in excess of net load.

Under a combined strategy, if the batteries cannot meet the net load, either a load-following or cycle-charging strategy is employed. With this strategy a critical load,  $L_c$ , is calculated for each scenario as follows:

$$L_c = \frac{8.415c_f}{c_{bw} + c_f \left( \frac{1}{\eta_{RT}} - 1 \right)}$$

$L_c$  is the net load where the cost of cycled diesel energy (denominator), i.e. that which is put into the battery by the generator and later extracted, is equal to the cost of direct diesel energy (numerator). For net loads less than  $L_c$ , it is more expensive on a \$/kWh basis to load-follow due to generator inefficiencies, thus the prescription is to cycle-charge and store any excess energy for later use. For net loads greater than  $L_c$ , diesel efficiencies are such that it is less expensive to load-follow. In this model, I assume the round trip efficiency,  $\eta_{RT}$ , is 1, so

$$L_c = \frac{8.415c_f}{c_{bw}} \quad (3.9)$$

An  $L_c$  is calculated for each simulated scenario according to equation 3.9 and is used in the combined strategy according to Rules 1 and 2.

Rule 1: if  $NL > soc$  and  $NL > L_c$  then load-follow

Rule 2: if  $NL > soc$  and  $NL < L_c$  then cycle-charge



Rule 1 states that if the charge in the batteries is not sufficient to meet the net load and the net load is greater than the critical load, then load-follow. Rule 2 states that if the charge in the batteries is not sufficient to meet the net load and the net load is less than the critical load, then cycle-charge.

Under the random strategy, realizations are drawn on a uniform distribution,  $U \sim [0,100]$ . This range represents all of the possible generator settings (rounded to nearest the action discretization step,  $\Delta a$ ). Since a random strategy is the most simplistic strategy possible, it serves as a useful baseline against which all other strategies can be compared.

In addition to using a simulation to test the optimal strategies, HOMER is used as a test platform. It should be noted that the model developed here is based on first principles of dispatch optimization and is a general framework. Consequently, its configuration is not fully compatible with the underlying assumptions contained in a specific model like HOMER. One such difference lies in the application of the loss of load penalty measured in dollars versus the "percent of unmet load" criterion used by HOMER.

In HOMER the percent of unmet load is an absolute constraint that screens out any strategies that exceed it. The percent unmet load is independent of any dispatch decision and is an indicator of what happened under a particular strategy. This is in contrast to the loss of load penalty in the general framework, which is a tool for making proactive dispatch decisions to avoid loss of load.

Another difference between the configuration of the model developed here and that of HOMER lies in the point at which a dispatch decision is made. The general framework assumes that the current state-of-charge and time-of-day capture all relevant information about the system up to the point in time where a dispatch decision needs to be made (see figure 7). The optimal strategies are proactive in that the decisions they make anticipate realizations of the net load over not only the next hour, but all subsequent hours as well. HOMER makes dispatch decisions as a reaction to the current net load only and assumes that the net load will be the same over the entire hour. Furthermore, HOMER's decisions are made regardless of net load beyond the current hour (see figure 8). Despite these configuration differences, the optimal dispatch strategies still do quite

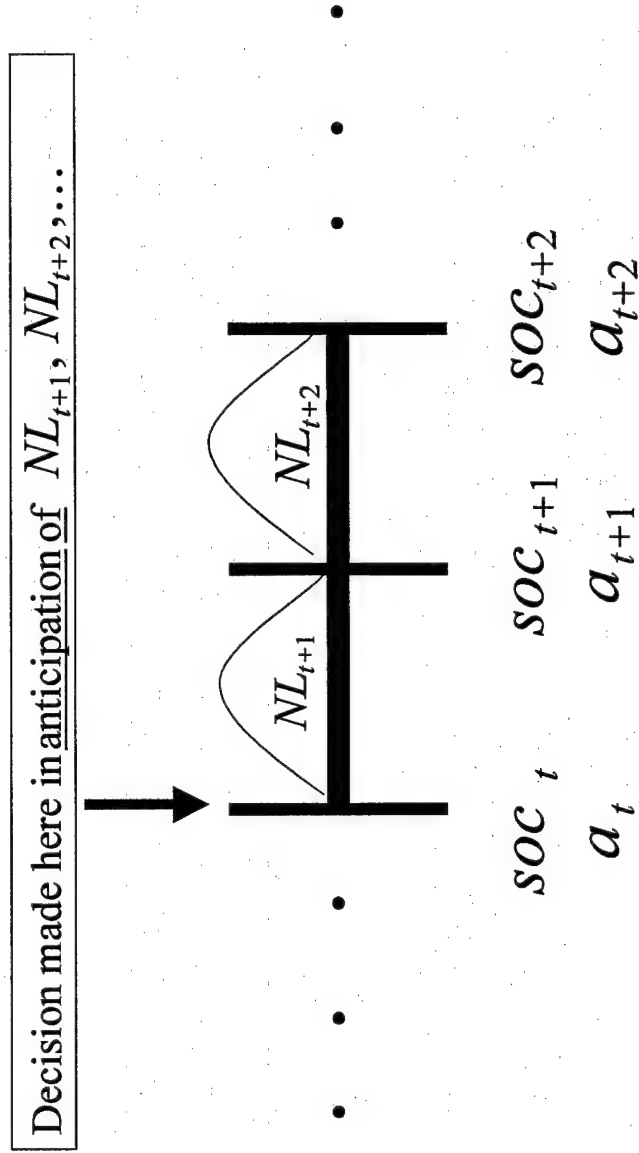


Figure 7. Optimal Dispatch Decision Process

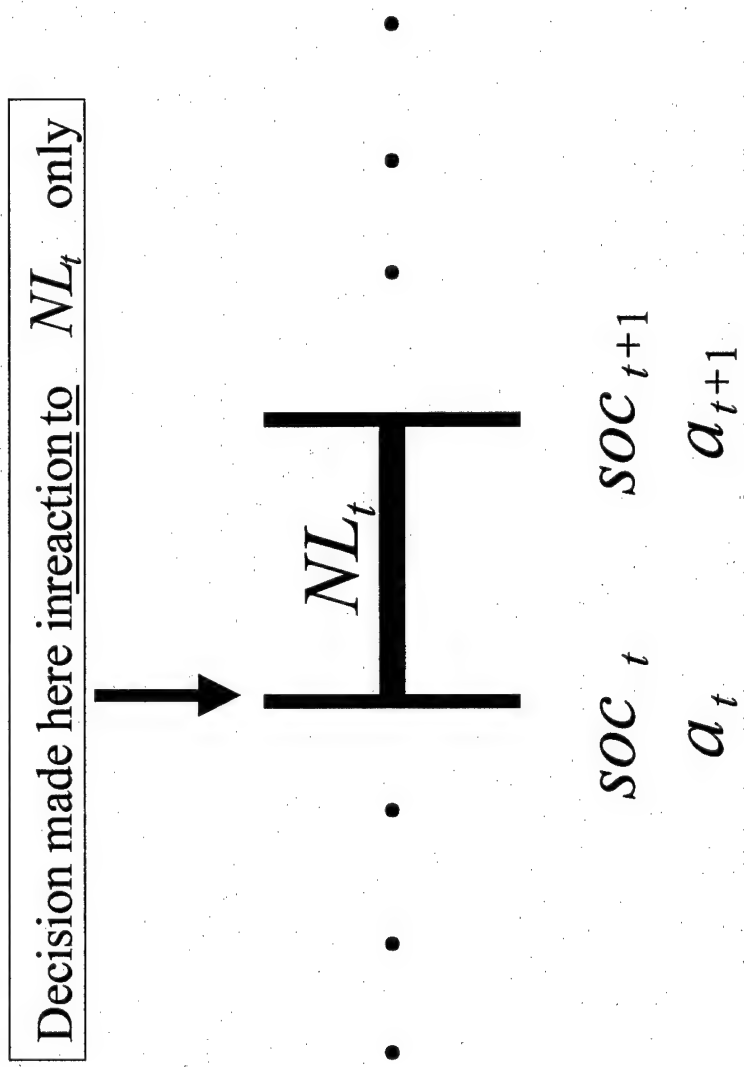


Figure 8. HOMER Decision Process

well when measured against standard dispatch strategies on a net present cost basis. This indicates potential for gains in cost reduction in a specific model like HOMER with appropriate configuration adjustments to the general framework.

## Chapter 4

## RESULTS AND ANALYSIS

4.1 Simulation Results

The solutions to the general framework provide state-values for each member of the state-space and a policy that maps each state to the action that minimizes the long-run average value or cost of each state. Examples of the output from the model are shown in figures 9 and 10. Figure 9 shows the discrete actions that should be taken for each

100	50	45	45	60	45	0	45	50	50	50	0	0	50	50	50	50	65	100	100	95	70	55	50	50
95	50	50	50	60	50	0	45	50	50	50	0	0	50	50	50	50	65	100	100	95	70	60	50	55
90	50	50	50	60	50	40	45	50	55	50	0	0	50	50	55	50	65	100	100	95	70	60	50	55
85	50	50	50	60	50	40	45	50	55	55	0	0	50	50	55	50	65	100	100	95	70	60	55	55
80	50	50	50	60	50	40	45	50	55	55	0	0	50	55	55	50	65	100	100	95	70	60	55	55
75	50	50	50	60	50	40	45	50	55	55	0	0	50	55	55	50	65	100	100	95	70	60	55	55
70	50	50	50	60	50	40	45	50	55	55	0	40	50	55	55	50	65	100	100	95	70	60	55	55
65	50	50	50	60	50	40	45	50	55	55	0	40	50	55	55	50	65	100	100	95	75	60	55	55
60	50	50	50	60	50	40	45	50	55	55	0	40	50	55	55	50	65	100	100	95	75	60	55	55
55	50	50	50	60	50	40	45	50	55	55	0	40	50	55	55	50	65	100	100	100	75	60	55	55
50	55	50	50	60	50	40	45	50	55	55	0	40	50	55	55	50	65	100	100	100	75	60	55	55
45	55	50	50	60	50	40	45	50	55	55	0	40	50	55	55	50	65	100	100	100	75	60	55	55
40	55	50	50	60	50	40	45	55	55	55	0	40	50	55	55	50	70	100	100	100	75	60	55	55
35	55	50	50	60	50	40	45	55	55	55	0	40	50	55	55	55	75	100	100	100	75	60	55	55
30	55	50	50	60	50	40	45	50	55	55	35	40	55	55	55	55	80	100	100	100	75	60	55	55
25	55	50	50	60	50	40	45	50	55	60	35	40	55	55	55	55	85	100	100	100	75	60	55	55
20	55	50	50	60	50	40	45	55	55	65	40	45	55	55	60	55	90	100	100	100	75	60	55	55
15	55	50	50	60	50	45	45	55	60	70	40	45	55	55	60	55	95	100	100	100	75	60	55	55
10	55	50	50	60	50	45	50	55	60	75	40	45	55	55	60	55	100	100	100	100	75	60	55	55
5	55	50	50	60	50	45	50	55	60	80	40	45	55	60	60	60	100	100	100	100	75	60	55	55
0	50	50	50	60	45	40	45	50	50	50	35	40	50	50	50	65	100	100	100	95	70	60	50	55
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23

Figure 9. Optimal Actions, Net Load Profile 1, Scenario 1

state-vector describing a combination of state-of-charge and time-of-day. Figure 10 is a 3-D representation of the same policy showing interpolated actions for each state not in the defined state-space. Interpolated actions are not used in this framework, so figure 10 is only intended to demonstrate potential for extending the results to a denser state-space. The optimal policies are tested against each of the other dispatch strategies described in chapter 3, section 7.

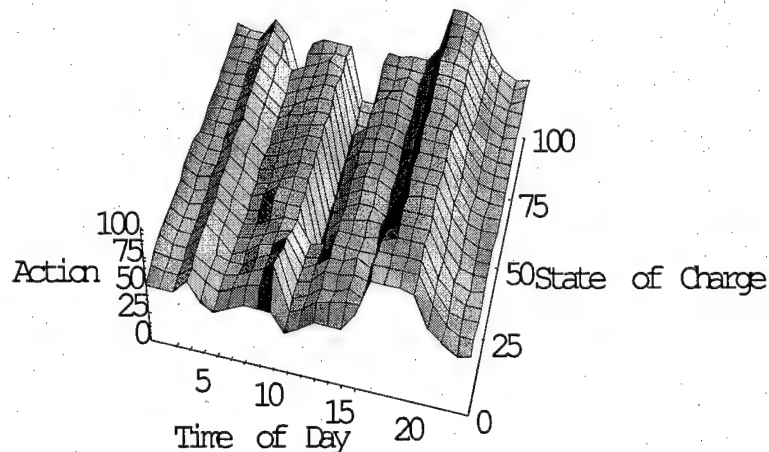


Figure 10. Optimal Action Profile, Net Load profile 1, Scenario 1

#### 4.1.1 Analysis: Net Load Profile 1

Figures 11 and 12 show a comparison of the long-run expected cost earned under each optimal dispatch strategy relative to the other dispatch strategies tested. It is clear from figure 11 that the optimal strategies under each scenario provide the lowest long-run expected cost compared to the other 4 strategies. Note Scenario 2 is not included in the comparisons. Since a loss of load penalty of \$0 was used in computing the optimal dispatch strategy, the model reported that the battery should drain fully and the generator should remain off. This is an intuitive result given the model has no incentive to minimize loss of load. Consequently, a long-run average cost of \$0 was reported which confirms the validity of the model's solution. Figure 12 is a summary of the results of the dispatch strategy comparison. GREEN cells represent the LOWEST VALUES under each scenario and RED cells represent the HIGHEST VALUES under each scenario. In all cases a low value is considered better than a high value since the goal of the optimization is cost minimization and minimization of loss of load.



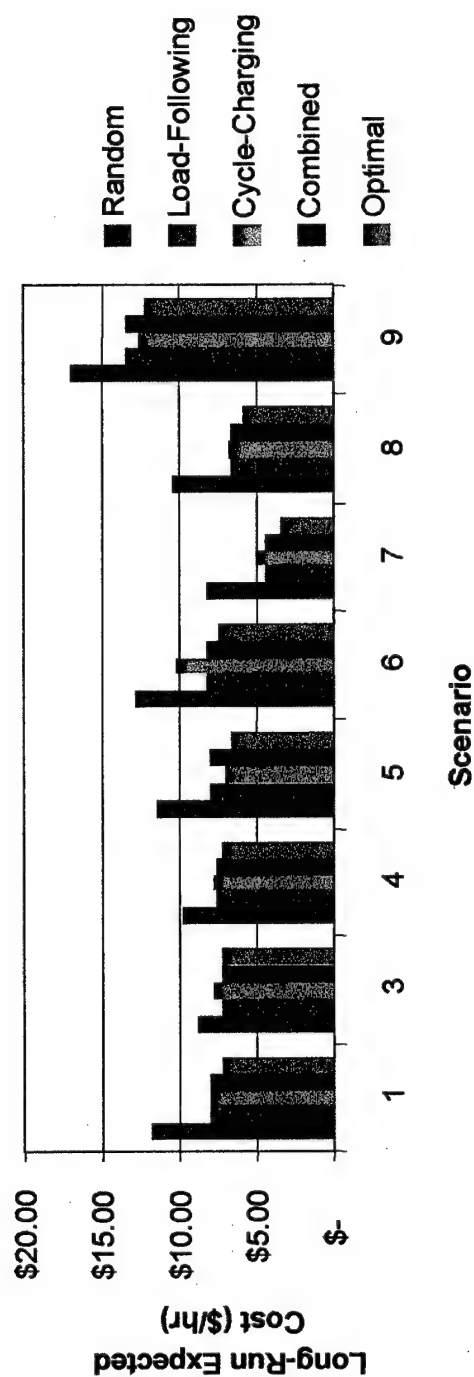


Figure 11. Comparison of Dispatch Strategies, Net Load Profile 1

Optimal	Scenario 1	Scenario 3	Scenario 4	Scenario 5	Scenario 6	Scenario 7	Scenario 8	Scenario 9
Expected Reward (\$/hr)	\$ 7.14	\$ 7.17	\$ 7.16	\$ 6.64	\$ 7.38	\$ 3.45	\$ 5.71	\$ 12.22
Total Reward Annual	\$ 62,546.40	\$ 62,809.20	\$ 62,721.60	\$ 58,166.40	\$ 64,648.80	\$ 30,222.00	\$ 50,019.60	\$ 107,047.20
Generator/Battery Cost	\$ 62,396.40	\$ 62,122.95	\$ 62,396.60	\$ 58,091.40	\$ 64,278.80	\$ 30,192.00	\$ 49,929.60	\$ 106,312.20
Loss	\$ 150.00	\$ 686.25	\$ 325.00	\$ 75.00	\$ 370.00	\$ 30.00	\$ 90.00	\$ 735.00

Load Following	Scenario 1	Scenario 3	Scenario 4	Scenario 5	Scenario 6	Scenario 7	Scenario 8	Scenario 9
Expected Reward (\$/hr)		\$ 7.27	\$ 7.54		\$ 8.20	\$ 4.42	\$ 6.64	
Total Reward Annual		\$ 63,685.20	\$ 66,050.40		\$ 71,832.00	\$ 38,719.20	\$ 58,166.40	
Generator/Battery Cost	\$ 61,320.60	\$ 61,363.95	\$ 61,407.90		\$ 62,547.00	\$ 29,434.20	\$ 48,881.40	\$ 107,398.20
Loss		\$ 2,321.25	\$ 4,642.50	\$ 9,285.00				\$ 9,285.00

Over-Dispatching	Scenario 1	Scenario 3	Scenario 4	Scenario 5	Scenario 6	Scenario 7	Scenario 8	Scenario 9
Expected Reward (\$/hr)				\$ 6.97				\$ 12.51
Total Reward Annual	\$ 67,720.60			\$ 61,057.20				\$ 109,587.60
Generator/Battery Cost	\$ 2,885.00	\$ 721.25	\$ 1,442.50	\$ 2,885.00	\$ 2,885.00	\$ 2,885.00	\$ 2,885.00	\$ 2,885.00
Loss								

Combined	Scenario 1	Scenario 3	Scenario 4	Scenario 5	Scenario 6	Scenario 7	Scenario 8	Scenario 9
Expected Reward (\$/hr)		\$ 7.27	\$ 7.54		\$ 8.20	\$ 4.42	\$ 6.64	
Total Reward Annual		\$ 63,685.20	\$ 66,050.40		\$ 71,832.00	\$ 38,719.20	\$ 58,166.40	
Generator/Battery Cost	\$ 61,320.60	\$ 61,363.95	\$ 61,407.90	\$ 59,490.00	\$ 62,547.00	\$ 29,434.20	\$ 48,881.40	\$ 106,083.20
Loss		\$ 2,321.25	\$ 4,642.50					

Figure 12. Summary of Dispatch Strategy Results, Net Load Profile 1

Recall that the long-run expected cost and associated annual costs include a combination of generator cost, battery wear costs, and loss of load penalties.

It is interesting to note that the optimal strategies all have the lowest overall costs, but may not have the lowest cost in each of the separate cost categories. The annual generator and battery wear costs that are incurred under the different scenarios are not necessarily the lowest under the optimal dispatch strategies.

For net load profile 1, the generator and battery wear costs are actually lower for the load-following and combined strategy under most of the scenarios. The optimal strategies sacrifice some generator and battery wear cost to keep loss of load to a minimum. Even so, the generator and battery wear costs for the optimal strategies are only marginally worse than the load-following and combined strategies. For optimal strategies the generator and battery wear costs are about 8-10% better than the worst case, cycle-charging strategy. Notice that for any non-zero loss of load penalty (Scenarios 1, 3, 4 with penalties of \$1, \$.25, \$.50, respectively and identical fuel and battery wear costs), the long-run average cost is

essentially the same for each scenario. This implies that any non-zero loss of load penalty will suffice, under this net load profile, to ensure that long-run average cost is kept to a minimum. The size of the loss of load cost and the associated quality of service are a function of the magnitude of the loss of load penalty. For net load profile 1, the optimal strategies happen to produce the lowest loss of load costs under all of the scenarios as well as provide the best level of service to the consumer, when compared to the other strategies.

In addition to loss of load cost, quality of service is analyzed using two other metrics. They are the number of hours in a year the system cannot meet demand and the average loss of load measured in kilowatts. For this net load profile, the optimal strategies are convincing winners in terms of these metrics (see figure 13). Under all of the scenarios, but one, the optimal strategies, have the fewest number of "outages" in a year with the lowest average loss of load measured in kW. For a \$1 penalty and any combination of fuel cost and battery wear cost, the optimal dispatch strategies for Scenarios 1 and 5 through 9 provide the fewest number of outages compared to all other

	Scenario 1	Scenario 3	Scenario 4	Scenario 5	Scenario 6	Scenario 7	Scenario 8	Scenario 9
Hours Lost Load per Year	29	499	121	15	70	6	18	146
% Lost Load Year (Hrs)	0.33%	5.60%	1.40%	0.17%	0.79%	0.07%	0.21%	0.16%
Average Loss (kW)	5.17	5.5	5.37	5	5.29	5	5	5.03
Loss Value (\$)	\$ 150.00	\$ 686.25	\$ 325.00	\$ 75.00	\$ 370.00	\$ 30.00	\$ 90.00	\$ 735.00

	Scenario 1	Scenario 3	Scenario 4	Scenario 5	Scenario 6	Scenario 7	Scenario 8	Scenario 9
Hours Lost Load per Year				954				954
% Lost Load Year (Hrs)				10.90%				10.90%
Average Loss (kW)	9.73	9.73	9.73	9.73	9.73	9.73	9.73	9.73
Loss Value (\$)				\$ 9,285.00				\$ 9,285.00

	Scenario 1	Scenario 3	Scenario 4	Scenario 5	Scenario 6	Scenario 7	Scenario 8	Scenario 9
Hours Lost Load per Year	229	229	229	229	229	229	229	229
% Lost Load Year (Hrs)	2.60%	2.60%	2.60%	2.60%	2.60%	2.60%	2.60%	2.60%
Average Loss (kW)								
Loss Value (\$)	\$ 2,885.00	\$ 721.25	\$ 1,442.50	\$ 2,885.00	\$ 2,885.00	\$ 2,885.00	\$ 2,885.00	\$ 2,885.00

	Scenario 1	Scenario 3	Scenario 4	Scenario 5	Scenario 6	Scenario 7	Scenario 8	Scenario 9
Hours Lost Load per Year								
% Lost Load Year (Hrs)								
Average Loss (kW)	9.73	9.73	9.73	10.55	9.73	9.73	9.73	10.55
Loss Value (\$)								

Figure 13. Quality of Service Analysis, Net Load Profile 1

strategies. Under Scenario 3, the loss of load penalty is \$.25/kW, so the incentive for the optimal strategies to minimize loss of load is reduced. Even so, the optimal strategy is not the worst in terms of quality of service under Scenario 3. Also, under Scenario 3, the average unmet load is far lower than the other strategies.

#### 4.1.2 Analysis: Net Load Profile 2

Figures 14 and 15 show a comparison of the long-run expected costs for each dispatch strategy under net load profile 2. The optimal strategies under each scenario provide the lowest long-run expected cost, but in some cases are only marginally better than standard dispatch strategies.

In particular, under Scenarios 5, 8 and 9, the combined strategy can provide essentially the same long-run expected cost as the optimal strategy. Note though, that under net load profile 2, the optimal strategy places more emphasis on reducing the generator and battery wear costs than the other strategies. In fact, the optimal strategies produce the

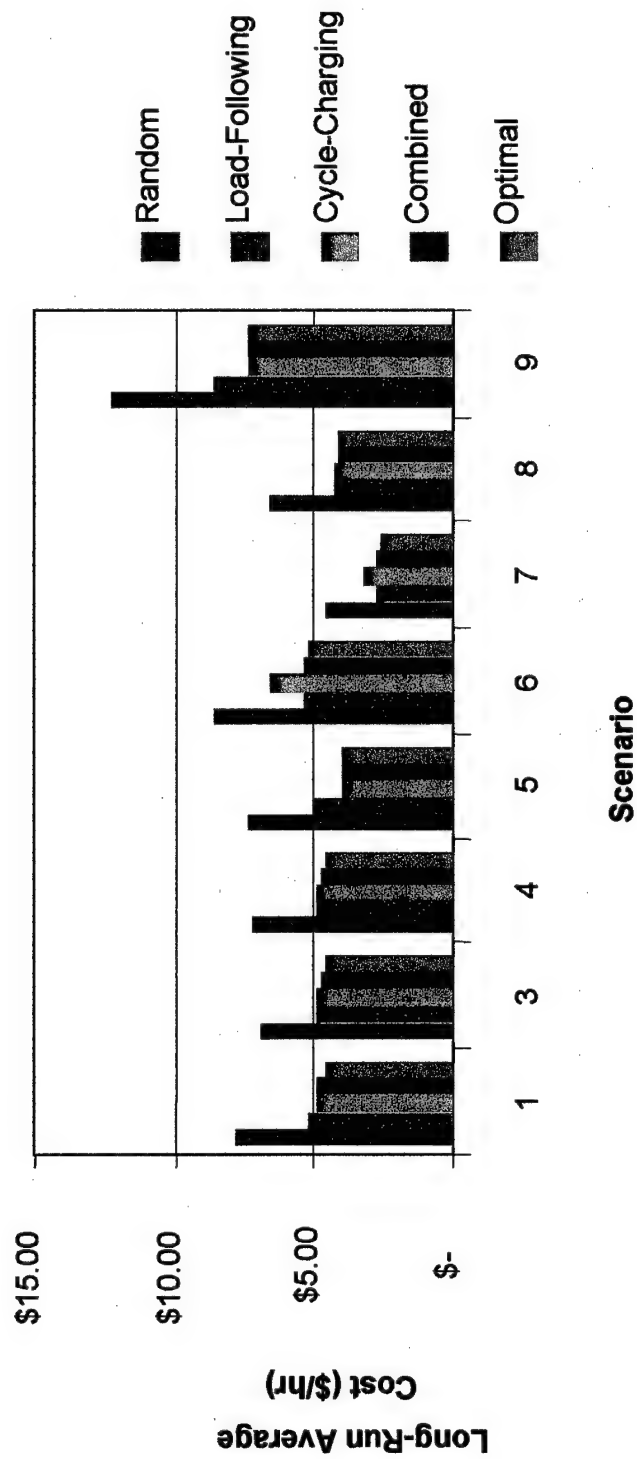


Figure 14. Comparison of Dispatch Strategies, Net Load Profile 2

Optimal	Scenario 1	Scenario 3	Scenario 4	Scenario 5	Scenario 6	Scenario 7	Scenario 8	Scenario 9
Expected Reward (\$/hr)	\$ 4.66	\$ 4.61	\$ 4.62	\$ 4.00	\$ 5.28	\$ 2.60	\$ 4.11	\$ 7.31
Total Reward	\$ 40,821.60	\$ 40,383.60	\$ 40,471.20	\$ 35,040.00	\$ 46,252.80	\$ 22,776.00	\$ 36,003.60	\$ 64,035.60
Generator/Battery Cost	\$ 39,261.60	\$ 38,947.35	\$ 39,133.70	\$ 34,370.00	\$ 41,522.80	\$ 20,041.00	\$ 32,043.60	\$ 62,745.60
Loss	\$ 1,560.00		\$ 1,337.50	\$ 670.00		\$ 2,735.00	\$ 3,960.00	\$ 1,290.00

Load Following	Scenario 1	Scenario 3	Scenario 4	Scenario 5	Scenario 6	Scenario 7	Scenario 8	Scenario 9
Expected Reward (\$/hr)					\$ 5.33	\$ 2.82		
Total Reward					\$ 46,690.80	\$ 24,703.20		
Generator/Battery Cost	\$ 41,469.40	\$ 41,487.00	\$ 41,451.60		\$ 42,695.80	\$ 20,708.20	\$ 33,410.20	
Loss		\$ 999.00			\$ 3,995.00			

Cycle Charging	Scenario 1	Scenario 3	Scenario 4	Scenario 5	Scenario 6	Scenario 7	Scenario 8	Scenario 9
Expected Reward (\$/hr)	\$ 4.89	\$ 4.84	\$ 4.85	\$ 4.04			\$ 4.21	\$ 7.34
Total Reward	\$ 42,836.40	\$ 42,398.40	\$ 42,486.00	\$ 35,390.40			\$ 36,879.60	\$ 64,298.40
Generator/Battery Cost				\$ 34,795.40				\$ 63,703.40
Loss	\$ 595.00	\$ 148.75	\$ 297.50	\$ 595.00	\$ 595.00	\$ 595.00	\$ 595.00	\$ 595.00

Combined	Scenario 1	Scenario 3	Scenario 4	Scenario 5	Scenario 6	Scenario 7	Scenario 8	Scenario 9
Expected Reward (\$/hr)	\$ 4.95	\$ 4.75	\$ 4.79	\$ 4.05	\$ 5.40	\$ 2.79	\$ 4.11	\$ 7.34
Total Reward	\$ 43,362.00	\$ 41,610.00	\$ 41,960.40	\$ 35,478.00	\$ 47,304.00	\$ 24,440.40	\$ 36,003.60	\$ 64,298.40
Generator/Battery Cost	\$ 41,027.00	\$ 41,026.25	\$ 40,792.90	\$ 34,838.00	\$ 44,969.00	\$ 21,770.40	\$ 33,668.60	\$ 63,658.40
Loss	\$ 2,335.00	\$ 583.75	\$ 1,167.50	\$ 640.00	\$ 2,335.00	\$ 2,670.00	\$ 2,335.00	\$ 640.00

Figure 15. Summary of Dispatch Strategy Results, Net Load Profile 2



lowest generator and battery wear costs over all of the scenarios (see figure 15).

Loss of load is not a criterion by which the standard strategies make dispatch decisions. While the optimal strategies are trying to balance generator and battery wear costs against loss of load penalties for an extended period of time; the other strategies simply do what is best to meet current load, regardless of the ramifications in subsequent time periods. For optimal strategies under this profile, the loss of load cost is generally somewhere between the best strategy for minimizing loss of load cost, cycle-charging and the worst strategy, load-following. Consequently, there are mixed results in terms of quality of service for the optimal strategies.

The optimal strategies, while less successful in avoiding loss of load costs, are generally not the worst strategies in terms of service quality (see figure 16). They do provide a mid-range level of service in most cases, but certainly do not provide as good a quality as the cycle-charging strategy. This makes sense in that the cycle-charging strategy always "tops off the batteries" and thus avoids some unexpected loss of load. This is an especially

	Scenario 1	Scenario 3	Scenario 4	Scenario 5	Scenario 6	Scenario 7	Scenario 8	Scenario 9
Hrs. Lost Load per Year	267	824	394	112		412	628	185
% Lost Load Year (Hrs)	3.00%	9.40%	4.50%	1.30%		4.70%	7.20%	2.10%
Avg rageloss (kW)		6.97	6.78	5.99				
Loss Value (\$)	\$ 1,560.00		\$ 1,337.50	\$ 670.00		\$ 2,735.00	\$ 3,960.00	\$ 1,290.00

	Scenario 1	Scenario 3	Scenario 4	Scenario 5	Scenario 6	Scenario 7	Scenario 8	Scenario 9
Hrs. Lost Load per Year		671	671		671			
% Lost Load Year (Hrs)		7.70%						
Avg rageloss (kW)	5.95	5.95	5.95	5.95	5.95	5.95	5.95	5.95
Loss Value (\$)		\$ 999.00			\$ 3,995.00			

	Scenario 1	Scenario 3	Scenario 4	Scenario 5	Scenario 6	Scenario 7	Scenario 8	Scenario 9
Hrs. Lost Load per Year	100	100	100	100	100	100	100	100
% Lost Load Year (Hrs)	1.14%	1.14%	1.14%	1.14%	1.14%	1.14%	1.14%	1.14%
Avg rageloss (kW)	5.95	5.95	5.95	5.95	5.95	5.95	5.95	5.95
Loss Value (\$)	\$ 595.00	\$ 148.75	\$ 297.50	\$ 595.00	\$ 595.00	\$ 595.00	\$ 595.00	\$ 595.00

	Scenario 1	Scenario 3	Scenario 4	Scenario 5	Scenario 6	Scenario 7	Scenario 8	Scenario 9
Hrs. Lost Load per Year	393			102	393	455	393	102
% Lost Load Year (Hrs)	4.50%			1.10%	4.50%	5.20%	4.50%	1.16%
Avg rageloss (kW)	5.94				5.94	5.87	5.94	6.27
Loss Value (\$)	\$ 2,335.00	\$ 583.75	\$ 1,167.50	\$ 640.00	\$ 2,335.00	\$ 2,670.00	\$ 2,335.00	\$ 640.00

Figure 16. Quality of Service Analysis, Net Load Profile 2

effective strategy for this net load profile since all realizations of net loads are below the battery capacity. Cycle-charging pays the price in terms of generator and battery wear costs, however. What the optimal strategies lack in service quality under this net load profile, they make up for in minimizing the generator and battery wear costs.

#### 4.2 HOMER Results

The results of using the optimal dispatch strategies in HOMER under net load profiles 1 and 2 are shown in tables 6 and 7, respectively. Since the loss of load penalty of \$1 is most effective in minimizing the loss of load in most circumstances, and HOMER is indifferent to a loss of load penalty, I only consider Scenarios 1 and 5 through 9.

Table 6 shows that for a net load profile with all positive net load realizations and a higher likelihood of loss of load, the optimal strategies have the lowest net present cost (NPC) and a low percentage of unmet load. Where a standard dispatch strategy does not appear indicates that its percent of unmet load is too large. The net

present cost reported by HOMER includes only generator and battery wear costs. The discounted costs reported by HOMER are calculated slightly differently than in the general framework, so the magnitudes are not reported, only the ranking of the strategies. The results in table 6 imply that the optimal strategies not only reduce loss of load over the life of the system, but also used the least amount of generator and battery resources. Notice that the percentage of unmet load is constant for all of the standard dispatch strategies over all of the scenarios. This is an indication that HOMER makes its dispatch decisions independently of (i.e. without anticipating) net load realizations; HOMER's decisions are based only on current net load. The percentage unmet load figures are different for the optimal strategies under each scenario. This is due to configuration differences between the general framework and HOMER. In the general framework, loss of load is anticipated and in some instances allowed if it leads to lower overall cost. A trade-off is made between the three components of cost, loss of load penalty, generator cost and battery wear cost, based on their respective parameters. For

Table 6. Results from HOMER: Net Load Profile 1

Scenario	Ranked by Lowest Net Present Cost	Percent Unserved Load
1	OP	.5%
	CC	.22%
	CB	.22%
	-	-
5	OP	0%
	-	-
	-	-
	-	-
6	OP	.48%
	CC	.22%
	CM	.22%
	-	-
7	OP	.37%
	CC	.22%
	CM	.22%
	-	-
8	OP	.26%
	CC	.22%
	CM	.22%
	-	-
9	OP	.07%
	-	-
	-	-
	-	-

OP-Optimal Strategy    CC-Cycle-Charge    CM-Combined    LF-Load-follow

Table 7. Results from HOMER: Net Load Profile 2

Scenario	Ranked by Lowest Net Present Cost	Percent Unserved Load
1	LF	0%
	CC	0%
	CM	0%
	OP	1.16%
5	OP	2.82%
	CC	0%
	CM	0%
	LF	0%
6	LF	0%
	OP	3.27%
	CC	0%
	CM	0%
7	LF	0%
	OP	.98%
	CC	0%
	CM	0%
8	LF	0%
	OP	2.5%
	CC	0%
	CM	0%
9	OP	2.9%
	CC	0%
	CM	0%
	LF	0%

OP-Optimal Strategy   CC-Cycle-Charge   CM-Combined   LF-Load-follow

net load profile 2 (table 7), the optimal strategies place either first or second in 5 of the 6 scenarios considered in HOMER. In the Scenario 1, where the optimal strategy placed last, there is only a 2% difference in the net present cost of running the system over a 20-year life. Notice that the percentage unmet load figures for the standard strategies are 0%. This is not a surprising result given that the net load realizations for this profile are all below the generator and battery capacities. The figures indicate that the system can at least meet load in every time step under the standard strategies. The optimal strategies show differing percentage unmet load because they attempt to balance the three components of cost in anticipation of net load realizations in the future. HOMER is indifferent to trade-offs between loss of load penalties and other costs, so it does not "credit" the optimal strategies with minimizing cost, only with minimizing net present cost.

#### 4.3 Implementing the General Optimization Framework

Whether or not one should use the general optimization framework presented here is obviously a function of the cost

of implementation. The following costs might be incurred in implementing the model:

- cost of gathering site data
- analysis of data to determine net load distributions
- training of analyst to familiarize him/her with the model
- computer run time
- periodic re-sampling of data to ensure the stationarity assumption still holds (see discussion in chapter 3.3.5)

In reality, the cost of implementing this framework should be minimal for the following reasons:

If we assume that a site has been selected for installation of a HRPS, it would be necessary to collect net load data for determining the proper mix of equipment to use. Therefore, the incremental cost of collecting site data for use in the general framework would be \$0 as it has already been collected. Furthermore, assuming some screening model, such as HOMER, is being used to field the system, an analysis of the net load distributions would also have been conducted. So, using the same argument as above, the incremental cost of analyzing the net load distributions



for use in the general framework would, again, be \$0. Calculation of the transition probabilities is automated in the model presented here. So, initially familiarizing an analyst with the workings of the model and specifying appropriate distribution scale and location figures, and cost parameters would take, in my estimation, approximately one and a one-half hours. This estimate came about because I explained how the model worked to Tom Lambert of NREL who integrated it into a research version of HOMER. It took him less than 45 minutes to understand the entire model. For safety I doubled this time to account for persons not of his level of understanding or education. Assuming an analyst makes \$20 per hour (Lilienthal 1999), this would amount to a cost estimate of \$30.

Computer run time for determining an optimal strategy, based on actual run times for 18 cases used in this work, is approximately 4 hours. If we assume the analyst already possesses a personal computer for doing site analysis, I would estimate that the incremental cost of calculating the optimal dispatch strategy might be as high as \$1. In total, the cost of initially setting up the system with an optimal

dispatch strategy, as assumed above, would be approximately \$31.

Periodic re-sampling of the site data would have to be conducted to ensure that the stationarity assumption still holds. Assuming that the system is being dispatched with heuristic methods, sensors would have to be available to tell the controller the current net load on the system. It is reasonable to assume that this sensor data is already being recorded since having such data would be useful in the future for fielding systems in villages of similar size or with similar weather conditions. Periodic re-sampling consists of: (1) determining the change in net load distributions, if any, (2) deciding if it is necessary to re-run the model, and (3) supplying the new data and parameters for the model. For re-sampling, I assume the analyst is already familiar with the model, so familiarization time is replaced by the time required to determine any net load distribution changes. In total, assume the incremental cost of re-sampling would be close to the \$31 estimated earlier. Clearly, the total annual cost of implementing optimal dispatch would be on the order of \$62, if we assume after an initial run that we re-sample

twice a year. It must be noted that re-sampling is required at some interval because of the possibility that usage of electricity would rise as people become accustomed to its availability.

Any conclusions regarding benefit-cost analyses based on the results presented here are limited to the net load profiles examined here and parameter specifications made in Scenarios 1 through 9. The benefit of using the optimal dispatch strategies is measured by the extent to which the optimal strategies reduce the long-run average cost per hour of running the system compared to the next best heuristic method (see table 8). Multiplying the hourly savings by

Table 8. Optimal Strategies vs. Next Best Heuristic

	Scenario							
	1	3	4	5	6	7	8	9
<b>Net Load Profile 1</b>								
Next Best Heuristic	\$8.06 (LF/CC/CM)	\$7.27 (LF/CM)	\$7.54 (LF/CM)	\$6.97 (LF/CM)	\$8.20 (LF/CM)	\$4.42 (LF/CM)	\$6.64 (LF/CM)	\$12.51 (LF/CM)
Optimal	\$7.14	\$7.77	\$7.16	\$6.64	\$7.38	\$3.45	\$5.71	\$12.22
<b>Net Load Profile 2</b>								
Next Best Heuristic	\$4.89 (CC)	\$4.70 (CC)	\$4.79 (CM)	\$4.04 (CC)	\$5.33 (LF)	\$2.79 (CM)	\$4.11 (CM)	\$7.34 (CM/CC)
Optimal	\$4.66	\$4.61	\$4.62	\$4.00	\$5.28	\$2.60	\$4.11	\$7.31

8,760 hours per year annualizes this benefit. The annual cost of implementing the optimal strategies is then netted against the annual cost savings to arrive at the annual net benefit of implementing the optimal strategies. These results are shown in tables 9 and 10. An explanation of tables 9 and 10 follows.

In table 9, the number in row 1, column 1 (\$.92) is the difference in long-run average cost between the next best heuristic method (LF/CC/CM--\$8.06) and the optimal dispatch strategy (\$7.14). Continuing down the first column in table 9, this hourly cost savings is annualized by multiplying \$.92 by 8,760 hours per year to arrive at an annual cost savings of \$8,059 under Scenario 1. The cost of implementation is then subtracted from \$8,059 to arrive at an annual net benefit of \$7,997. The annual net benefit is inflated at a rate of 3% per year (Survey of Professional Forecasters 1999) over a typical 20-year system life. The inflated annual net benefits are then discounted using discount rates ranging from 5% to 20%.

Under net load profile 1 (table 9), the annual net benefit of using the optimal dispatch strategies versus using the next best heuristic strategy in a system ranges

from \$814 to \$8,435. Under all discount rates ranging from 5% to 20%, the net present benefit over the 20-year life of the system is positive under all scenarios. This implies that it makes sense to use the optimal strategies for dispatching systems facing a net load profile similar to

Table 9. Benefit-Cost Analysis, Net Load Profile 1

Net Load Profile 1		Scenario						
	1	3	4	5	6	7	8	9
Cost								
Savings (\$/hr)	\$0.92	\$0.10	\$0.38	\$0.33	\$0.82	\$0.97	\$0.93	\$0.29
Annual Cost Savings	\$8,059	\$876	\$3,329	\$2,891	\$7,183	\$8,497	\$8,147	\$2,540
Cost of Implementation/Maintenance	\$62	\$62	\$62	\$62	\$62	\$62	\$62	\$62
Annual Net Benefit	\$7,997	\$814	\$3,267	\$2,829	\$7,121	\$8,435	\$8,085	\$2,478
Discount Rate								
5%	\$134,057	\$13,645	\$54,761	\$47,419	\$119,373	\$141,399	\$135,526	\$41,545
10%	\$83,935	\$8,543	\$34,287	\$29,690	\$74,741	\$88,532	\$84,854	\$26,012
15%	\$60,185	\$6,126	\$24,585	\$21,289	\$53,593	\$63,481	\$60,844	\$18,652
20%	\$45,794	\$4,661	\$18,707	\$16,198	\$40,778	\$48,302	\$46,296	\$14,192

that of net load profile 1. Under net load profile 2 (table 10), the annual net benefit of using the optimal dispatch strategies in a system ranges from -\$62 to \$1,953. This implies that there may be circumstances, Scenario 8 for example, where it is more advantageous to use an existing heuristic dispatch method. Under this scenario the cost of

implementing the optimal strategy exceeds any cost savings that might be realized. Nevertheless, under the other 7 scenarios, a net annual benefit produces a positive net present benefit of using the optimal strategies over the life of the system. So, again, it makes sense to use the optimal dispatch strategies in those seven scenarios for systems facing net load profiles comparable to net load profile 2.

Table 10. Benefit-Cost Analysis, Net Load Profile 2

Net Load Profile 2	Scenario							
	1	3	4	5	6	7	8	9
Cost								
Savings (\$/hr)	\$0.23	\$0.14	\$0.17	\$0.04	\$0.05	\$0.19	\$0.00	\$0.03
Annual Savings	\$2,015	\$1,226	\$1,489	\$350	\$438	\$1,664	\$0	\$263
Cost of Implementation/ Maintenance	\$62	\$62	\$62	\$62	\$62	\$62	\$62	\$62
Annual Net Benefit	\$1,953	\$1,164	\$1,427	\$288	\$376	\$1,602	-\$62	\$201
Discount Rate								
5%	\$32,797	\$19,581	\$23,986	\$4,896	\$6,365	\$26,923	-\$977	\$3,428
10%	\$20,496	\$12,221	\$14,979	\$3,027	\$3,946	\$16,818	-\$651	\$2,108
15%	\$14,696	\$8,763	\$10,741	\$2,170	\$2,830	\$12,059	-\$467	\$1,511
20%	\$11,182	\$6,668	\$8,173	\$1,651	\$2,153	\$9,176	-\$355	\$1,150

#### 4.4 Conclusions

Using a relatively parsimonious model, the results of the simulations show that under the net load profiles

considered here the optimal dispatch strategies minimize the long-run average and total costs under all scenarios. It has been shown that optimal dispatch strategies can reduce total system life-cycle cost where generator cost, battery wear and loss of load penalties are considered.

For net load profiles, such as net load profile 1, where loss of load is almost assured, the optimal strategies shine. They minimize loss of load while incurring only marginally more generator and battery wear cost than the other strategies. This has a further advantage in that it implies it might be possible to use an undersized generator (one that is not sized to meet expected peak demand) in a HRPS. This would reduce initial capital costs as larger generators are more expensive. For net load profiles where a wide range of net load realizations can be expected from one hour to the next, the optimal strategies incur more loss of load in exchange for reducing generator and battery wear costs.

The results of running the optimal strategies in HOMER differ from those of the simulation. This is to be expected given that the simulation is an exact depiction of the conditions under which the general framework was developed.

Even though the configuration of HOMER is not true to the general framework, there is enough evidence to suggest that modifications to the framework can achieve improvements in the way HOMER or any other specific HRPS model makes dispatch decisions.

By extending the state-vector in the model to include current net load as an element, any advantages of the optimal strategies have over the standard strategies will only be magnified.

This model simultaneously considers generator costs, battery wear costs and loss of load penalties in dispatch decisions where other models do not. It also gives the user the flexibility to balance service availability against lower operating costs.

Dispatch decisions can be made independently of initial and final conditions of the system, avoiding potentially sub-optimal decisions about battery charge levels or generator settings at the beginning or end of a planning period. Also, in the event of a system stoppage and regardless of the experience of the operator, optimal dispatch decisions can be made manually.



The framework is predictive in nature, anticipating potential net load realizations in future periods and the ramifications of any immediate dispatch decisions. It is also flexible enough to consider any non-linearities in cost or constraint functions.

## Chapter 5

### ISSUES FOR FURTHER ANALYSIS

Now that it has been established that optimal strategies can reduce long-run average cost, an interesting extension to this work would be to determine the types of net load profiles over which the optimal dispatch strategies are most effective. Here, one fairly standard net load profile (Net Load Profile 1) and another with fairly unusual characteristics (Net Load Profile 2) were used. Clearly, the optimal strategies are more effective with Net Load Profile 1 in reducing long-run average cost relative to the standard strategies than they are with Net Load Profile 2. It would be interesting to record the variance of the net load profiles around the profile mean and the variance of the net loads around their respective hourly means, then map these characteristics against the optimal strategies and the standard strategies. This would require the analysis of several more net load profiles and require perhaps hundreds of hours more of simulation time.

Under the assumptions used here, the state-of-charge and time-of-day capture all of the relevant information up to the point in time where an action must be taken. A further extension of the model would expand the state-vector to include current net load. Thus, the model would not only anticipate future net load realizations, but it would react to current system requirements as a separate entity rather than include it as one realization out of a set of net loads expected over the next time period.

The disadvantage of expanding the state-vector and consequently the state-space is that the number of states grows as the product of the state-of-charge, time-of-day and the number of new elements in the current net load descriptor. So, if the current net load descriptor contains only 4 elements for example, which is an optimistically low number, the size of the state-space would grow to  $504 \times 4 = 2,016$  states. Then solution algorithms such as Policy Iteration would take considerably longer on the desktop computer platform used here. Other solution algorithms would have to be considered, such as modified policy iteration, reinforcement learning or other mechanisms described by Rust (1996).

Another extension would include an appropriate allowance for non-linearities in battery wear. When batteries are charged and discharged from a high state of charge, they experience less "wear." However, it is advantageous to periodically use most of the charge in the battery and then fully charge it again. Conversely, as batteries are charged and discharged from low state of charge, they experience a great deal of wear. Also, they should not be left in low states of charge for extended periods of time. All of these issues could be incorporated into the cost structure of this model.

The HRPS considered here are intended to provide power for village size populations. The concept could be extended to providing power to small, remote military bases. Most remote military bases use generator power to meet essential and non-essential power requirements. In the event that fuel availability is strictly curtailed or eliminated altogether, a HRPS could be used to meet essential power requirements.

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Appendix A  
DESCRIPTION OF THE HYBRID OPTIMIZATION  
MODEL FOR ELECTRIC RENEWABLES (HOMER)

Until the advent of HOMER either relatively simple spreadsheet models or very complex simulation models were used for the preliminary design of hybrid renewable power systems (Lilienthal 1995, 1). This hindered the development of HRPS because no simple methods for determining the sizing of the components existed. Spreadsheet models, useful for financial analysis, cannot reliably predict the performance of hybrid systems with intermittent resources and storage because they do not consider the temporal patterns of loads and resources. Dynamic simulation models are useful for evaluating the performance of specific, well-defined systems, but are very detailed and time consuming to set up and run.

HOMER fills the gap between these two types of models by giving the user insight into how the system design is affected by changes in parameters, such as component costs,



technology performance, load management strategy, dispatch strategies, and load and resource profiles. Since the optimal size of each component is an output of HOMER and the detailed simulation models require the user specify the size of each component, there is a natural flow of information between the models. When considering a new site, HOMER can quickly determine a small set of optimal or near-optimal systems for more detailed analysis with simulation models.

HOMER requires two basic types of input. The first input describes hourly profiles of the available renewable resources and loads and the second describes the components of the system (Lilienthal 1995).

Resources and loads must be specified hourly. Although actual hourly data can be used, a more compact representation uses hourly profiles for typical days for each season. A season can be any number of consecutive weeks. In order to model the stochastic nature of both renewable resources and loads, these typical days are then expanded into a user-specified number of modeled days by adding two types of noise to the profiles. An hourly noise parameter independently perturbs each of the hourly values, while a daily noise parameter perturbs an entire day. The

daily noise is important because persistent weather patterns have a major impact on storage requirements. For resource profiles, HOMER uses Normal distributions that incorporate both the variation inherent in the typical profile and the two noise parameters. In addition to the typical profiles and noise parameters, average wind speed and average full-sun hours are specified for each season. Separate residential and commercial loads (electrical demands) can be specified for weekdays and weekends.

Five types of components must be described for HOMER: wind turbines, photovoltaic arrays, batteries, inverters, and diesel generators. Capital cost and lifetime are specified for each component. Normally, HOMER chooses the optimal size of each component, but each of these sizes can be fixed if a particular system is being modeled. All components, except the wind turbines, are continuous variables. Wind turbines come in discrete units, so HOMER chooses the number of each type of turbine. PV has a fixed cost as well as a cost per kW. Efficiencies are specified for the inverter and diesel generators. The diesel parameters can be specified separately for small and large diesels to capture the economies of scale in that

technology. Diesel operation and maintenance is specified as a cost per hour of operation.

Batteries require special consideration when modeling isolated systems. HOMER keeps track of their state-of-charge and energy flow in each hour, subject to constraints on the rate at which they can be charged and discharged as well as their minimum acceptable state-of-charge. Furthermore, the lifetime of a battery is a function of usage, so HOMER charges depreciation for each kWh of battery throughput.

HOMER outputs include hourly energy flows through each component, the impact of several dispatch strategies, and economic information such as the cost of energy and net present cost of the system.

## Appendix B

## CODE FOR POLICY ITERATION ALGORITHM

**Bolded Lines are Comments**

```
Public Const numhours = 24, numcharge = 21, numactions = 21,
epsilon = 0.1, beta = 0.9999, pr = 100, cf = 0.264, res = 5,
losspen = 5, battcost = 0.1
```

```
numhours=number of hours;
numactions=number of actions available;
epsilon=acceptable tolerance for state value
beta= discount factor;
pr=max power rating of generator;
cf=cost of fuel per liter;
res=resolution of state of charge and actions
losspen= loss of load penalty;
battcost=battery wear cost
```

```
Public v(1 To numcharge, 1 To numhours), pol(1 To numcharge,
1 To numhours), numstates, a(numactions), cost, valuestate(1
To (numhours * numcharge)), polycystate(1 To (numhours *
numcharge)), timestate(1 To (numhours * numcharge)), p, k,
socstate(1 To (numhours * numcharge)), penalty, nl, wear,
myprob, loss, dump, OM, diesel, iter, polycystable,
countload(0 To numhours - 1), netload(0 To numhours - 1, 1
To 100), probnetload(0 To numhours - 1, 1 To 100), ps, first
```

```
!*****
```

```
Sub Stochastic()
```

```
first = False
```

```
Is this the first
run with these
parameters ?
```

```

numstates = numhours * numcharge

starttime = Time

iter = 0

!*****

If res = 10 Then
    Call load10
ElseIf res = 5 Then
    Call load5
Else:
    Call load2
End If

!*****

Call initialactions

If first = True Then
    Call initialstatepolicy

```

False means use  
prior runs results  
as seeds for the  
next run

Total number of  
states in state  
space

Marks starting time

Initialize policy  
eval and improvement  
counter to zero

Load netloads and  
probabilities based on  
resolution being used

Main Subroutine

```

Else: Call mreload
End If

line10: Call policyeval
iter = iter + 1
Call policyimprove

If polycystable = False Then
    GoTo line10
End If

*****

For i = 1 To numcharge
    For j = 1 To numhours
        Worksheets("Policy").Cells(i, j + 2) =
            polycystate((i - 1) * numhours + j)
    Next j
Next i

For i = 1 To numcharge
    For j = 1 To numhours
        Worksheets("Values").Cells(i, j + 2) =
            valuestate((i - 1) * numhours + j)
    Next j
Next i

```

Reload old  
values if this  
is NOT first  
run

Solves Bellman  
Equations

Checks for  
better policies  
for each state

Print out policy  
and values

```

endtime = Time
Worksheets("Values").Cells(1, 1) = starttime
Worksheets("Values").Cells(2, 1) = endtime

End Sub

'*****

Sub initialactions()
For m = 1 To numactions
    a(m) = (m - 1) * res
Next m
End Sub

'*****

Sub initialstatepolicy()
For i = 1 To numcharge
    soc = ((100 / res) + 1 - i) * res
    For j = 1 To numhours
        t = (j - 1)
        v(i, j) = 0
        pol(i, j) = a(1)
        k = ((i - 1) * numhours) + j

```

**Marks ending time**

**Assigns action set to a(m)**

**Initializes policy and values if first run**

**Calculates SOC for this state**

**Assigns initial values and policies**

**Calculates time for this state**

**Creates scalar equivalents**

```

        valuestate(k) = v(i, j)

        polycystate(k) = pol(i, j)
        timestate(k) = t

        socstate(k) = soc

    Next j

Next i

End Sub

'*****

Sub mreload()

    For i = 1 To numcharge

        For j = 1 To numhours

            polycystate((i - 1) * numhours + j) =
                Worksheets("Policy").Cells(i, j + 2)

        Next j

    Next i

    For i = 1 To numcharge

        For j = 1 To numhours

            valuestate((i - 1) * numhours + j) =
                Worksheets("Values").Cells(i, j + 2)

```

**Assigns values  
and policies to  
scalars**

**Assigns soc and  
time to scalar  
states**

**Uses values and  
policies from  
previous run  
as seeds**

**Load old policy and  
values into current  
state space**



```

    Next j
Next i
For i = 1 To numcharge
    soc = ((100 / res) + 1 - i) * res           Calculates soc
                                                for this state

    For j = 1 To numhours
        t = (j - 1)                             Calculates time
                                                for this state

        k = ((i - 1) * numhours) + j           Creates scalar
                                                equivalents

        timestate(k) = t                       Assigns SOC and
                                                time to scalar
                                                states

        socstate(k) = soc

    Next j
Next i
End Sub

'*****

Sub policyeval()                               Solves Bellman
                                                equations

    Dim delta

    Open "c:\p_eval.txt" For Output As #2 (opens file for data
    storage)

    Do
        delta = 0

        For k = 1 To numstates

```

```

tempvalue = valuestate(k)
storevalue = 0
storeprob = 0
checkmod = k Mod numhours

If k < numhours And checkmod <> 0 Then
    For p = (k + 1) To ((k + 1) + numhours
        * (numcharge - 1)) Step numhours

        Call calcprob
        Call calc cost

        storeprob = storeprob + myprob
        storevalue = storevalue + myprob * (cost + beta *
            valuestate(p))

    If (k = 99) Then Print #2, "s " & k & " s' " & p & " prob "
        & myprob & " cost " & cost & " loss " & loss & " expnl " &
        nl & " wear " & wear & " value " & storevalue

    Writes data to file for analysis

    Application.StatusBar = "eval iter= " & iter + 1 & " k= " &
        k & " p= " & p & "a= " & a(m)

    Shows status of algorithm

    Next p

ElseIf checkmod = 0 Then
    For p = 1 To (numhours * (numcharge - 1) + 1)
        Step numhours

```

**For each state  
save current value  
of state k**

**Top row**

**Calculates  
transition  
probabilities**

**Next possible  
successor state**

**Last column**

```

Call calcprob
Call calccost

storeprob = storeprob + myprob
storevalue = storevalue + myprob * (cost + beta
    * valuestate(p))

If (k = 99) Then Print #2, "s " & k & " s' " & p & " prob "
& myprob & " cost " & cost & " loss " & loss & " expnl " &
nl & " wear " & wear & " value " & storevalue

Application.StatusBar = "eval iter= " & iter + 1 & " k= " &
k & " p= " & p & "a= " & a(m)

Next p

Else

Not top row
or last
column

For p = (timestate(k) + 2) To ((numcharge - 1) *
    numhours + (timestate(k) + 2)) Step numhours

Call calcprob

Call calccost

storeprob = storeprob + myprob
storevalue = storevalue + myprob * (cost + beta
    * valuestate(p))

If (k = 99) Then Print #2, "s " & k & " s' " & p & " prob "
& myprob & " cost " & cost & " loss " & loss & " expnl " &
nl & " wear " & wear & " value " & storevalue

Application.StatusBar = "eval iter= " & iter + 1 & " k= " &
k & " p= " & p & "a= " & a(m)

Next p

End If

```

```
If Abs(storeprob - 1) > 0.000001 And Abs(storeprob - 1) <> 1
```

```
Then Stop
```

**Checks  
probabilities  
add to 1**

```
End If
```

```
valuestate(k) = storevalue
```

```
If Abs(tempvalue - valuestate(k)) > delta Then
```

```
delta = Abs(tempvalue - valuestate(k))
```

**Stopping rule for solving Bellman equs  
under existing policies**

```
Else: delta = delta
```

```
End If
```

```
Next k
```

```
Loop Until delta < epsilon
```

```
Application.StatusBar = False
```

```
Turns off status bar
```

```
Close #2
```

```
End Sub
```

```

*****

Sub policyimprove()

'Open "c:\pol_imp" For Output As #1

    policystable = True

    For k = 1 To numstates

        b = policystate(k)
        betterpolicy = policystate(k)
        previousvalue = -1000000000
        checkmod = k Mod numhours

        For m = 1 To numactions

            storevalue = 0

            policystate(k) = a(m)
            storeprob = 0

        If k < numhours And checkmod <> 0 Then

            For p = (k + 1) To ((k + 1) + numhours * (numcharge - 1))
                Step numhours

            Application.StatusBar = "policyiter iter= " & iter & " k= "
            & k & " action=" & a(m) & " p= " & p

```

Checks for better policies for each state

Assume current policy is best

For each state save current policy for state k

Stores value under each policy

Top row

```

Call calcprob
Call calccost

```

```

storeprob = storeprob + myprob
storevalue = storevalue + myprob * (cost + beta *
    valuestate(p))

```

```

If (k = 99) Then Print #1, "s " & k & " s' " & p & " prob "
& myprob & " cost " & cost & " loss " & loss & " expnl " &
nl & " wear " & wear & " value " & storevalue

```

```

Next p

```

```

ElseIf checkmod = 0 Then

```

```

Last
Column

```

```

For p = 1 To (numhours * (numcharge - 1) + 1) Step
numhours

```

```

Application.StatusBar = "policyiter iter= " & iter & " k= "
& k & " action=" & a(m) & " p= " & p

```

```

Call calcprob
Call calccost

```

```

storeprob = storeprob + myprob
storevalue = storevalue + myprob * (cost + beta *
    valuestate(p))

```

```

If (k = 99) Then Print #1, "s " & k & " s' " & p & " prob "
& myprob & " cost " & cost & " loss " & loss & " expnl " &
nl & " wear " & wear & " value " & storevalue

```

```

Next p

```

```

Else

```

```

Not top
row or

```

last  
column

```
For p = (timestate(k) + 2) To ((numcharge - 1) *  
    numhours + (timestate(k) + 2)) Step numhours
```

```
Application.StatusBar = "policyiter iter= " & iter & " k= "  
& k & " action=" & a(m) & " p= " & p
```

```
Call calcprob  
Call calccost
```

```
storeprob = storeprob + myprob  
storevalue = storevalue + myprob * (cost + beta *  
    valuestate(p))
```

```
If (k = 99) Then Print #1, "s " & k & " s' " & p & " prob "  
& myprob & " cost " & cost & " loss " & loss & " expnl " &  
n1 & " wear " & wear & " value " & storevalue
```

```
Next p
```

```
End If
```

```
If (storevalue >= previousvalue) Then
```

```
    previousvalue = storevalue  
    betterpolicy = a(m)
```

```
End If
```

```
Next m
```

```
policystate(k) = betterpolicy
```

```
If b <> policystate(k) Then
```

```
    policystable = False
```

```
End If
```

```
Next k
```

**Application.StatusBar = False**

**Close #1**

**End Sub**

**\*\*\*\*\***

**Sub calcprob()**

**Calculates  
transition  
probabilities**

**If (timestate(p) = timestate(k) + 1) Or  
    (timestate(p) = timestate(k) - (numhours - 1))**

**Then**

**pt = 1**

**Else: pt = 0**

**End If**

**If socstate(p) > 0 And socstate(p) < 100 Then**

**Call probstoch1**

**Ending state is  
in middle of  
state space**

**myprob = pt \* ps**

**ElseIf socstate(p) = 100 And (socstate(k) + policystate(k) -  
    netload(timestate(p), 1)) >= 100**

**Then**

**Call probstoch2**

**Ending state is  
at top of state  
space**

**myprob = pt \* ps**

**ElseIf (socstate(p) = 0) And (socstate(k) + policystate(k) -  
    netload(timestate(p), countload(timestate(p)))) <= 0**

**Then**



```

    Call probstoch3
    myprob = pt * ps
Else
    myprob = 0
End If
End Sub

'*****

Sub probstoch1()
    100>SOC(p)>0
    For w = 1 To countload(timestate(p))
        If (socstate(k) + policystate(k) - socstate(p)) <>
            netload(timestate(p), w) Then
            ps = 0
        Else
            ps = probnetload(timestate(p), w)
            GoTo line20
        End If
    Next w
line20: End Sub

'*****

Sub probstoch2()
    SOC(p)=100
    tempprob = 0
    For x = 1 To countload(timestate(p)) Step 1

```

Ending state is  
at bottom of  
state space

```
If ((socstate(k) + policystate(k) - netload(timestate(p),
x)) >= socstate(p)) Then
```

```
    tempprob = tempprob + probnetload(timestate(p), x)
    ps = tempprob
```

```
Else
```

```
    GoTo line40
```

```
End If
```

```
Next x
```

```
line40: End Sub
```

```
!*****
```

```
Sub probstoch3()
```

```
SOC(p)=0
```

```
    tempprob = 0
```

```
    tempnl = 0
```

```
    For x = countload(timestate(p)) To 1 Step -1
```

```
        If ((socstate(k) + policystate(k) - netload(timestate(p),
x)) <= 0) Then
```

```
            tempprob = tempprob + probnetload(timestate(p), x)
            tempnl = tempnl + netload(timestate(p), x) *
            probnetload(timestate(p), x)
```

```
            ps = tempprob
```

```
            nl = tempnl
```

```
        Else
```

```
            GoTo line30
```

```
        End If
```

```
    Next x
```

```
line30: End Sub
```

!\*\*\*\*\*

Sub calccost()

**Calculates reward**

If polycystate(k) = 0 Then

diesel = 0

OM = 0

End If

If polycystate(k) <> 0 Then

diesel = -((0.08415 \* pr + 0.246 \* polycystate(k)) \* cf)

**Diesel cost**

OM = -1

End If

If socstate(k) > socstate(p) Then

wear = -((socstate(k) - socstate(p)) \* battcost)

**Battery wear cost**

Else: wear = 0

End If

If (socstate(p) = 0) And (socstate(k) + polycystate(k) - nl) <= 0 Then

loss = -(nl - socstate(k) - polycystate(k)) \*

losspen

**Loss of load penalty**

Else: loss = 0

End If

cost = diesel + loss + OM + wear

**Total immediate reward**

End Sub

!\*\*\*\*\*

Sub load2()

**Load  
resolution 2  
netloads**

For i = 1 To 24

    countload(i - 1) = Worksheets("netloadprobs2").Cells(5,  
    1 + i)

Next i

For i = 1 To 24

    For j = 1 To countload(i - 1)

        netload(i - 1, j) =  
        Worksheets("netloadprobs2").Cells(6 + j, 1 + i)

        probnetload(i - 1, j) =  
        Worksheets("netloadprobs2").Cells(68 + j, 1 + i)

    Next j

End Sub

!\*\*\*\*\*

Sub load5()

**Load  
resolution 5  
netloads**

For i = 1 To 24

    countload(i - 1) = Worksheets("netloadprobs5").Cells(5,

```

        1 + i)
Next i
For i = 1 To 24
    For j = 1 To countload(i - 1)
        netload(i - 1, j) =
        Worksheets("netloadprobs5").Cells(6 + j, 1 + i)

        probnetload(i - 1, j) =
        Worksheets("netloadprobs5").Cells(33 + j, 1 + i)

    Next j
Next i

End Sub

'*****

Sub load10()
    Load
    resolution
    10 netloads

    For i = 1 To numhours
        countload(i - 1) = Worksheets("netloadprobs10").Cells(5,
        1 + i)
    Next i

    For i = 1 To numhours
        For j = 1 To countload(i - 1)
            netload(i - 1, j) =
            Worksheets("netloadprobs10").Cells(6 + j, 1 + i)

            probnetload(i - 1, j) =
            Worksheets("netloadprobs10").Cells(23 + j, 1 + i)

```

Next j

Next i

End Sub

Appendix C  
OPTIMAL DISPATCH CHARTS

State of Charge (kWh)	Time of Day																							
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
100	50	45	45	60	45	0	45	50	50	50	0	0	50	50	50	50	65	100	100	95	70	55	50	50
95	50	50	50	60	50	0	45	50	50	50	0	0	50	50	50	50	65	100	100	95	70	60	50	55
90	50	50	50	60	50	40	45	50	55	50	0	0	50	50	55	50	65	100	100	95	70	60	50	55
85	50	50	50	60	50	40	45	50	55	55	0	0	50	50	55	50	65	100	100	95	70	60	55	55
80	50	50	50	60	50	40	45	50	55	55	0	0	50	55	55	50	65	100	100	95	70	60	55	55
75	50	50	50	60	50	40	45	50	55	55	0	0	50	55	55	50	65	100	100	95	70	60	55	55
70	50	50	50	60	50	40	45	50	55	55	0	40	50	55	55	50	65	100	100	95	70	60	55	55
65	50	50	50	60	50	40	45	50	55	55	0	40	50	55	55	50	65	100	100	95	75	60	55	55
60	50	50	50	60	50	40	45	50	55	55	0	40	50	55	55	50	65	100	100	95	75	60	55	55
55	50	50	50	60	50	40	45	50	55	55	0	40	50	55	55	50	65	100	100	100	75	60	55	55
50	55	50	50	60	50	40	45	50	55	55	0	40	50	55	55	50	65	100	100	100	75	60	55	55
45	55	50	50	60	50	40	45	50	55	55	0	40	50	55	55	50	65	100	100	100	75	60	55	55
40	55	50	50	60	50	40	45	55	55	55	0	40	50	55	55	50	70	100	100	100	75	60	55	55
35	55	50	50	60	50	40	45	55	55	55	0	40	50	55	55	55	75	100	100	100	75	60	55	55
30	55	50	50	60	50	40	45	50	55	55	35	40	55	55	55	55	80	100	100	100	75	60	55	55
25	55	50	50	60	50	40	45	50	55	60	35	40	55	55	55	55	85	100	100	100	75	60	55	55
20	55	50	50	60	50	40	45	55	55	65	40	45	55	55	60	55	90	100	100	100	75	60	55	55
15	55	50	50	60	50	45	45	55	60	70	40	45	55	55	60	55	95	100	100	100	75	60	55	55
10	55	50	50	60	50	45	50	55	60	75	40	45	55	55	60	55	100	100	100	100	75	60	55	55
5	55	50	50	60	50	45	50	55	60	80	40	45	55	60	60	60	100	100	100	100	75	60	55	55
0	50	50	50	60	45	40	45	50	50	50	35	40	50	50	50	65	100	100	100	95	70	60	50	55

Figure 17. Optimal Strategy, Net Load Profile 1, Scenario 1



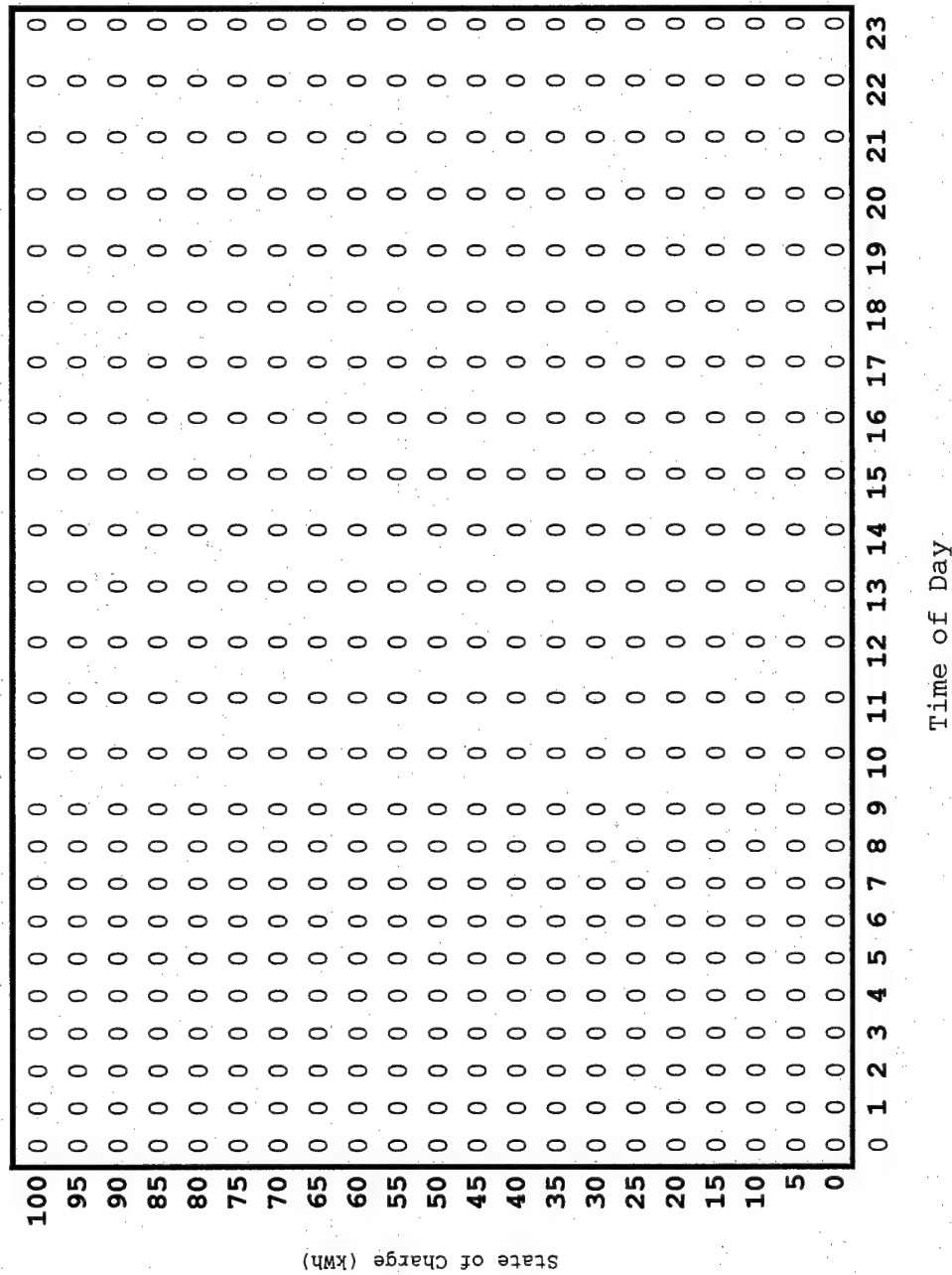


Figure 18. Optimal Strategy, Net Load Profile 1, Scenario 2

State of Charge (kWh)	Time of Day																							
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
100	50	45	45	60	45	0	45	50	50	50	0	0	50	50	50	50	65	100	100	95	70	55	50	50
95	50	50	50	60	45	0	45	50	50	50	0	0	50	50	55	50	65	100	100	95	70	60	50	55
90	50	50	50	60	50	40	45	50	55	55	0	0	50	50	55	50	65	100	100	95	70	60	50	55
85	50	50	50	60	50	40	45	50	55	55	0	0	50	50	55	50	65	100	100	95	70	60	55	55
80	50	50	50	60	50	40	45	50	55	55	0	0	50	55	55	50	65	100	100	95	70	60	55	55
75	50	50	50	60	50	40	45	50	55	50	0	0	50	55	55	50	65	100	100	95	70	60	55	55
70	50	50	50	60	50	40	45	50	55	50	0	0	50	55	55	50	65	100	100	95	70	60	55	55
65	50	50	50	60	50	40	45	50	55	50	0	0	50	55	55	50	65	100	100	100	75	60	55	55
60	50	50	50	60	50	40	45	50	55	50	0	0	50	55	55	50	65	100	100	100	75	60	55	55
55	50	50	50	60	50	40	45	50	55	55	0	0	50	55	55	50	65	100	100	100	75	60	55	55
50	50	50	50	60	50	40	45	50	55	55	0	0	50	55	55	50	65	100	100	100	75	60	55	55
45	50	50	50	60	50	40	45	50	55	55	0	40	50	50	55	50	65	100	100	100	75	60	55	55
40	50	50	50	60	50	40	45	50	55	55	0	0	50	50	55	50	65	100	100	100	75	60	55	55
35	50	50	50	60	50	40	45	50	55	50	0	40	50	50	55	50	70	100	100	100	75	60	55	55
30	55	50	50	60	50	40	45	50	55	55	0	40	50	55	55	50	70	100	100	100	75	60	55	55
25	55	50	50	60	50	40	45	50	55	60	35	40	50	55	55	55	75	100	100	100	75	60	55	55
20	55	50	50	60	50	40	45	55	55	65	35	40	55	55	60	55	80	100	100	100	75	60	55	55
15	55	50	50	60	50	45	50	55	60	70	40	45	55	55	55	60	85	100	100	100	75	60	55	55
10	55	50	50	60	50	45	50	60	65	75	40	45	55	55	60	65	90	100	100	100	75	60	55	55
5	55	50	50	60	50	45	50	65	60	80	40	45	55	55	60	70	95	100	100	95	70	60	55	55
0	50	50	50	60	45	40	45	50	50	50	35	40	50	50	50	50	100	100	100	95	70	55	50	55

Figure 19. Optimal Strategy, Net Load Profile 1, Scenario 3



State of Charge (kWh)	Time of Day																								
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	
100	0	0	0	55	0	0	0	0	0	0	50	0	0	0	0	50	0	65	100	100	95	70	55	0	0
95	0	0	0	60	0	0	0	0	0	0	50	0	0	0	0	55	0	65	100	100	95	70	60	0	55
90	0	55	0	60	0	0	0	0	0	0	55	0	0	55	0	55	0	70	100	100	100	75	65	0	60
85	0	60	0	65	0	0	0	0	0	0	60	0	0	55	0	60	0	75	100	100	100	80	70	0	65
80	0	65	0	70	0	0	0	0	0	0	65	0	0	60	0	60	0	80	100	100	100	85	75	0	70
75	0	70	0	75	0	0	0	0	0	0	70	0	0	65	0	60	0	85	100	100	100	90	80	0	75
70	0	75	0	80	0	0	0	0	0	75	75	0	0	70	0	65	0	90	100	100	100	95	85	0	80
65	0	80	0	85	0	0	0	0	0	80	80	0	0	75	0	70	0	95	100	100	100	100	90	0	85
60	0	85	0	90	0	0	0	0	0	85	85	0	0	80	0	75	0	100	100	100	100	100	95	0	90
55	90	90	0	95	85	0	0	0	0	90	90	0	0	85	0	80	0	80	100	100	100	80	100	60	95
50	95	95	0	100	90	0	0	0	0	95	95	0	0	0	0	85	95	85	100	100	100	80	100	0	100
45	100	100	95	100	95	0	0	100	100	100	0	0	95	95	90	100	90	90	100	100	100	85	100	60	100
40	100	100	100	100	100	0	55	100	100	100	0	0	100	100	95	100	95	100	100	100	100	90	100	65	100
35	65	100	100	100	100	100	60	100	65	100	0	100	100	100	100	100	100	100	100	100	100	95	100	70	100
30	70	100	100	100	100	100	65	100	70	100	90	60	100	60	100	80	100	80	100	100	100	100	100	75	100
25	75	100	80	100	100	100	70	100	75	100	95	65	100	65	100	85	100	85	100	100	100	100	80	100	100
20	80	100	85	100	100	100	75	100	80	100	100	70	100	70	100	90	100	90	100	100	100	100	85	85	100
15	85	100	90	100	75	100	80	100	85	100	100	75	100	75	100	95	100	95	100	100	100	100	90	90	100
10	90	100	95	100	80	100	85	85	90	100	100	80	100	80	100	100	100	100	100	100	100	100	95	95	100
5	95	100	100	95	85	100	90	90	95	85	100	85	100	85	100	100	100	100	100	100	100	100	100	100	100
0	100	100	100	100	90	100	95	50	100	90	100	90	100	90	50	85	100	100	100	100	100	100	60	100	100

Figure 21. Optimal Strategy, Net Load Profile 1, Scenario 5

State of Charge (kWh)	Time of Day																								
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	
100	50	50	50	60	45	40	45	50	50	50	50	35	40	50	50	55	50	65	100	100	95	70	60	55	55
95	50	50	50	60	45	40	45	50	50	50	50	35	40	50	50	55	50	65	100	100	95	70	60	55	55
90	50	50	50	60	45	40	45	50	50	50	50	35	40	50	50	55	50	65	100	100	95	70	60	55	55
85	50	50	50	60	45	40	45	50	50	50	50	35	40	50	50	55	50	65	100	100	95	70	60	55	55
80	50	50	50	60	45	40	45	50	50	50	50	35	40	50	50	55	50	65	100	100	95	70	60	55	55
75	50	50	50	60	45	40	45	50	50	50	50	35	40	50	50	55	50	65	100	100	95	70	60	55	55
70	50	50	50	60	45	40	45	50	50	50	50	35	40	50	50	55	50	65	100	100	95	70	60	55	55
65	50	50	50	60	45	40	45	50	50	50	50	35	40	50	50	55	50	65	100	100	95	70	60	55	55
60	50	50	50	60	45	40	45	50	50	50	50	35	40	50	50	55	50	65	100	100	95	70	60	55	55
55	50	50	50	60	45	40	45	50	50	50	50	35	40	50	50	55	50	65	100	100	95	70	60	55	55
50	50	50	50	60	45	40	45	50	50	50	50	35	40	50	50	55	50	65	100	100	95	70	60	55	55
45	50	50	50	60	45	40	45	50	50	50	50	35	40	50	50	55	50	65	100	100	95	70	60	55	55
40	50	50	50	60	45	40	45	50	50	50	50	35	40	50	50	55	50	65	100	100	95	70	60	55	55
35	50	50	50	60	45	40	45	50	50	50	50	35	40	50	50	55	50	70	100	100	95	70	60	55	55
30	50	50	50	60	45	40	45	50	50	50	50	35	40	50	50	55	55	75	100	100	95	70	60	55	55
25	50	50	50	60	50	40	45	50	50	50	50	35	40	50	55	55	55	80	100	100	95	70	60	55	55
20	50	50	50	60	50	40	45	50	50	50	50	35	40	50	55	55	60	85	100	100	95	70	60	55	55
15	50	50	50	60	50	40	45	50	50	50	50	45	55	55	55	60	55	90	100	100	95	70	60	55	55
10	50	50	50	60	50	40	45	50	50	50	50	45	55	55	60	60	55	95	100	100	95	70	60	55	55
5	50	50	50	60	50	40	45	50	50	50	50	45	55	55	60	60	55	100	100	100	95	70	60	55	55
0	50	50	50	60	45	40	45	50	50	50	50	35	40	50	50	50	60	100	100	100	95	70	60	50	55

Figure 22. Optimal Strategy, Net Load Profile 1, Scenario 6

		Time of Day																								
		0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	
100	50	50	50	50	60	45	40	45	50	50	50	50	35	40	50	50	55	50	65	100	100	95	70	60	55	55
95	50	50	50	50	60	45	40	45	50	50	50	50	35	40	50	50	55	50	65	100	100	95	70	60	55	55
90	50	50	50	50	60	45	40	45	50	50	50	50	35	40	50	50	55	50	65	100	100	95	70	60	55	55
85	50	50	50	50	60	45	40	45	50	50	50	50	35	40	50	50	55	50	65	100	100	95	70	60	55	55
80	50	50	50	50	60	45	40	45	50	50	50	50	35	40	50	50	55	50	65	100	100	95	70	60	55	55
75	50	50	50	50	60	45	40	45	50	55	50	55	35	40	50	50	55	50	65	100	100	95	70	60	55	55
70	50	50	50	50	60	45	40	45	50	55	50	55	35	40	50	50	55	50	65	100	100	95	70	60	55	55
65	50	50	50	50	60	45	40	45	50	55	50	55	35	40	50	50	55	50	65	100	100	95	70	60	55	55
60	50	50	50	50	60	45	40	45	50	55	50	55	35	40	50	50	55	50	65	100	100	95	70	60	55	55
55	50	50	50	50	60	50	40	45	50	55	50	55	35	40	50	50	55	50	65	100	100	95	70	60	55	55
50	50	50	50	50	60	50	40	45	50	55	50	55	35	40	50	50	55	50	65	100	100	95	70	60	55	55
45	50	50	50	50	60	50	40	45	50	55	50	55	35	40	50	50	55	50	65	100	100	95	70	60	55	55
40	50	50	50	50	60	50	40	45	50	55	50	55	35	40	50	55	55	50	70	100	100	95	70	60	55	55
35	50	50	50	50	60	50	40	45	50	55	50	55	35	40	50	55	55	50	75	100	100	95	70	60	55	55
30	50	50	50	50	60	50	40	45	50	55	50	55	35	40	50	55	55	55	80	100	100	95	70	60	55	55
25	50	50	50	50	60	50	40	45	50	55	55	35	40	55	55	55	55	55	85	100	100	95	70	60	55	55
20	50	50	50	50	60	50	40	45	50	55	55	40	45	55	55	55	60	55	90	100	100	95	70	60	55	55
15	50	50	50	50	60	50	40	45	50	55	55	40	45	55	55	55	60	55	95	100	100	95	75	60	55	55
10	50	50	50	50	60	50	40	45	50	55	55	40	45	55	60	60	55	100	100	100	95	75	60	55	55	
5	50	50	50	50	60	50	40	45	50	55	55	40	45	55	60	60	60	100	100	100	95	70	60	55	55	
0	50	50	50	60	45	40	45	50	50	50	50	35	40	50	50	50	65	100	100	100	95	70	60	50	55	

Figure 23. Optimal Strategy, Net Load Profile 1, Scenario 7

	Time of Day																							
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
100	50	50	50	50	60	45	40	45	50	50	50	0	40	50	50	50	65	100	100	95	70	60	50	55
95	50	50	50	50	60	45	40	45	50	50	50	0	40	50	50	55	65	100	100	95	70	60	50	55
90	50	50	50	50	60	45	40	45	50	55	50	0	40	50	50	55	65	100	100	95	70	60	50	55
85	50	50	50	50	60	45	40	45	50	55	50	0	40	50	50	55	65	100	100	95	70	60	55	55
80	50	50	50	50	60	50	40	45	50	55	50	0	40	50	55	55	65	100	100	95	70	60	55	55
75	50	50	50	50	60	50	40	45	50	55	50	0	40	50	55	55	65	100	100	95	70	60	55	55
70	50	50	50	50	60	50	40	45	50	55	50	0	40	50	55	55	65	100	100	95	70	60	55	55
65	50	50	50	50	60	50	40	45	50	55	50	0	40	50	55	55	65	100	100	95	70	60	55	55
60	50	50	50	50	60	50	40	45	50	55	55	35	40	50	55	55	65	100	100	95	70	60	55	55
55	50	50	50	50	60	50	40	45	50	55	55	35	40	50	55	55	65	100	100	100	75	60	55	55
50	50	50	50	50	60	50	40	45	50	55	55	35	40	50	55	55	65	100	100	95	75	60	55	55
45	50	50	50	50	60	50	40	45	50	55	55	35	40	50	55	55	65	100	100	100	75	60	55	55
40	50	50	50	50	60	50	40	45	50	55	55	35	40	50	55	55	70	100	100	100	75	60	55	55
35	50	50	50	50	60	50	40	45	50	55	55	35	40	50	55	55	75	100	100	100	75	60	55	55
30	50	50	50	50	60	50	40	45	50	55	55	35	40	50	55	55	80	100	100	100	75	60	55	55
25	50	50	50	50	60	50	40	45	50	55	55	35	40	55	55	60	85	100	100	100	75	60	55	55
20	50	50	50	50	60	50	40	45	50	55	55	40	45	55	55	65	90	100	100	100	75	60	55	55
15	50	50	50	50	60	50	40	45	50	55	55	40	45	55	60	70	95	100	100	100	75	60	55	55
10	50	50	50	50	60	50	40	45	50	55	55	40	45	60	65	55	100	100	100	95	75	60	55	55
5	50	50	50	50	60	50	40	45	50	55	55	40	45	65	70	60	100	100	100	95	70	60	55	55
0	50	50	50	50	60	45	40	45	50	50	50	35	40	50	50	65	100	100	100	95	70	60	50	55

State of Charge (kWh)

Figure 24. Optimal Strategy, Net Load Profile 1, Scenario 8

State of Charge (kWh)	Time of Day																								
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	
100	0	0	0	55	0	0	0	0	0	50	0	0	0	0	0	50	0	65	100	100	95	70	55	0	0
95	0	0	0	60	0	0	0	0	0	50	0	0	0	0	0	55	0	65	100	100	95	70	60	0	55
90	0	0	0	60	0	0	0	0	0	55	0	0	0	0	0	55	0	70	100	100	100	75	65	0	60
85	60	60	0	65	0	0	0	0	0	55	0	0	0	0	0	55	0	65	100	100	100	75	65	0	65
80	65	65	0	70	55	0	0	0	0	60	0	0	0	0	0	60	0	65	100	100	100	80	70	0	70
75	70	0	0	75	60	0	0	0	60	65	0	0	0	0	0	60	0	70	100	100	100	80	75	0	75
70	75	0	0	80	65	0	0	0	65	70	0	0	0	0	0	60	0	70	100	100	100	80	80	0	80
65	80	0	0	85	70	0	0	0	70	75	0	0	0	0	0	60	0	70	100	100	100	80	85	0	85
60	85	0	0	90	75	0	0	0	75	80	0	0	0	0	0	60	0	70	100	100	100	80	90	0	90
55	90	0	0	95	80	0	0	0	80	85	0	0	0	0	0	65	0	70	100	100	100	75	60	0	95
50	0	0	0	100	85	0	0	0	85	90	0	0	0	0	0	0	55	70	100	100	100	70	60	0	100
45	100	100	60	100	90	0	0	95	60	95	0	0	100	55	75	55	70	100	100	100	75	65	60	100	
40	100	100	65	100	95	0	55	100	60	100	0	0	100	60	80	55	70	100	100	100	75	70	65	65	
35	65	100	70	65	100	100	60	100	60	100	0	100	90	65	85	55	75	100	100	100	80	75	70	70	
30	70	100	75	70	100	100	65	100	65	100	95	60	95	70	90	55	80	100	100	100	85	80	75	75	
25	75	75	80	75	100	100	70	100	70	100	100	65	100	75	95	60	85	100	100	100	80	85	80	80	
20	80	80	85	80	100	100	75	55	75	100	100	70	100	80	100	55	90	100	100	100	80	90	85	85	
15	85	85	90	85	75	100	80	55	80	100	100	75	100	85	100	55	95	100	100	100	80	95	90	90	
10	90	90	95	90	80	100	85	55	85	80	100	80	90	90	100	55	100	100	100	100	80	100	95	95	
5	95	95	100	95	85	80	90	55	90	85	95	85	95	95	100	60	100	100	100	100	80	100	100	100	
0	100	100	100	100	90	85	95	50	95	90	100	90	100	100	100	50	65	100	100	100	95	70	60	50	

Figure 25. Optimal Strategy, Net Load Profile 1, Scenario 9



State of Charge (kWh)	Time of Day																							
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
100	0	0	0	0	0	0	0	0	40	0	0	0	0	0	0	0	60	60	60	80	80	40	0	0
95	0	0	0	0	0	0	0	40	45	0	0	0	0	0	0	0	60	60	60	80	80	40	0	0
90	0	0	0	0	0	45	45	45	45	0	0	0	0	0	0	0	65	60	65	80	80	45	0	0
85	0	0	0	0	0	45	45	45	50	35	0	0	0	0	0	0	65	60	65	80	80	45	0	0
80	0	0	0	0	0	45	45	45	50	40	45	0	0	0	0	0	65	60	65	80	80	45	0	0
75	0	0	0	0	0	45	45	50	55	45	50	0	0	35	0	0	65	60	65	85	85	45	0	0
70	0	0	0	0	0	45	45	50	60	50	55	0	0	35	0	0	65	65	70	85	85	45	0	0
65	0	0	0	0	0	45	50	50	65	55	60	0	0	0	0	0	70	60	70	85	85	45	0	0
60	0	0	0	0	0	45	50	55	70	0	0	0	0	0	0	0	65	60	65	85	85	45	0	0
55	0	0	0	0	0	45	50	55	75	65	70	0	0	0	0	0	65	60	65	85	85	45	0	0
50	0	0	0	0	0	50	50	55	80	70	75	45	0	0	0	0	65	60	65	85	85	50	0	0
45	0	0	0	0	0	50	50	60	85	75	80	50	0	0	35	0	65	60	65	85	90	50	0	0
40	0	0	0	0	0	50	55	60	90	80	85	55	0	0	35	0	70	65	70	90	85	50	0	0
35	0	0	0	0	0	50	55	60	95	85	90	60	0	0	35	0	70	60	70	85	85	45	0	0
30	0	0	0	0	0	50	55	60	100	90	0	0	0	0	0	0	65	60	70	85	85	50	0	0
25	0	0	0	0	0	50	55	65	100	95	100	70	100	65	35	30	65	60	65	85	85	50	0	0
20	0	0	0	0	0	50	55	65	80	100	100	75	100	70	40	30	65	60	65	85	85	55	0	0
15	0	0	0	0	0	55	60	65	85	75	75	80	45	75	45	35	70	65	70	90	90	60	20	0
10	0	0	0	0	0	55	60	70	90	80	80	85	50	80	50	40	70	60	70	95	95	65	0	15
5	0	0	0	0	0	35	35	75	95	85	85	90	55	85	55	45	75	65	75	100	100	70	30	15
0	10	0	0	10	5	40	40	40	35	90	90	95	60	90	60	30	60	60	60	80	80	75	35	20

Figure 26. Optimal Strategy, Net Load Profile 2, Scenario 1



State of Charge (kWh)	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	
100	0	0	0	0	0	0	0	0	40	0	0	0	0	0	0	0	0	60	60	60	80	80	40	0	0
95	0	0	0	0	0	0	0	40	45	0	0	0	0	0	0	0	0	60	60	60	80	80	40	0	0
90	0	0	0	0	45	45	45	45	45	0	0	0	0	0	0	0	0	65	60	65	80	80	45	0	0
85	0	0	0	0	45	45	45	45	50	35	0	0	0	0	0	0	0	65	60	65	80	80	45	0	0
80	0	0	0	0	45	45	45	45	50	40	45	0	0	0	0	0	0	65	60	65	80	80	45	0	0
75	0	0	0	0	45	45	50	55	55	45	50	0	0	0	0	0	0	65	60	65	85	85	45	0	0
70	0	0	0	0	45	45	50	60	60	50	55	0	0	35	0	0	0	65	65	70	90	85	45	0	0
65	0	0	0	0	45	50	50	60	60	55	60	0	0	0	0	0	0	70	70	70	85	85	45	0	0
60	0	0	0	0	45	50	55	65	65	0	0	0	0	0	0	0	0	75	60	65	85	85	45	0	0
55	0	0	0	0	45	50	55	70	65	70	70	0	0	0	0	0	0	65	60	65	85	85	45	0	0
50	0	0	0	0	50	50	55	75	70	75	75	45	0	0	0	0	0	65	60	65	85	85	45	0	0
45	0	0	0	0	50	50	60	80	75	80	80	50	0	0	0	0	0	65	60	65	85	85	45	0	0
40	0	0	0	0	0	50	60	85	80	85	85	55	0	0	35	0	65	60	65	85	85	45	0	0	
35	0	0	0	0	45	0	0	0	85	90	60	0	0	0	0	0	65	60	65	85	85	45	0	0	
30	0	0	0	0	45	50	50	95	90	0	0	0	0	0	0	0	65	60	65	80	80	45	0	0	
25	0	0	0	0	50	50	55	100	95	100	70	95	65	35	0	65	60	65	80	80	45	0	0	0	
20	0	0	0	0	55	55	55	80	100	100	75	100	70	40	30	65	60	65	85	85	50	0	0	0	
15	0	0	0	0	55	60	60	85	75	75	80	45	75	45	30	65	65	70	90	85	55	0	0	0	
10	0	0	0	0	60	65	65	90	80	80	85	50	80	50	35	70	60	70	95	90	60	0	0	0	
5	0	0	0	0	35	35	35	95	85	85	90	55	85	55	30	65	65	75	100	95	65	0	0	0	
0	0	0	0	0	40	40	35	35	90	90	95	60	90	60	30	60	60	60	80	80	70	0	0	0	

Figure 28. Optimal Strategy, Net Load Profile 2, Scenario 3

State of Charge (kWh)	Time of Day																								
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	
100	0	0	0	0	0	0	0	0	40	0	0	0	0	0	0	0	0	60	60	60	80	80	40	0	0
95	0	0	0	0	0	0	0	40	45	0	0	0	0	0	0	0	0	60	60	60	80	80	40	0	0
90	0	0	0	0	0	45	45	45	45	0	0	0	0	0	0	0	0	65	60	65	80	80	45	0	0
85	0	0	0	0	0	45	45	45	50	35	0	0	0	0	0	0	0	65	60	65	80	80	45	0	0
80	0	0	0	0	0	45	45	45	50	40	45	0	0	0	0	0	0	65	60	65	85	80	45	0	0
75	0	0	0	0	0	45	45	50	55	45	50	40	0	0	0	0	0	65	60	65	85	85	45	0	0
70	0	0	0	0	0	45	45	50	60	50	55	45	0	35	0	0	0	65	60	65	90	85	45	0	0
65	0	0	0	0	0	45	50	50	65	55	60	0	0	0	0	0	0	65	60	65	85	85	45	0	0
60	0	0	0	0	0	45	50	55	70	0	0	0	0	0	0	0	0	65	60	65	85	85	45	0	0
55	0	0	0	0	0	45	50	55	75	65	70	0	0	0	0	0	0	65	60	65	85	85	45	0	0
50	0	0	0	0	0	50	50	55	80	70	75	45	0	0	0	0	0	65	60	65	85	85	45	0	0
45	0	0	0	0	0	50	50	60	85	75	80	50	80	0	0	0	0	65	60	65	85	85	45	0	0
40	0	0	0	0	0	50	55	60	90	80	85	55	85	0	35	0	0	65	60	65	85	85	45	0	0
35	0	0	0	0	0	50	50	0	0	85	90	60	0	0	40	0	0	65	60	65	85	85	45	0	0
30	0	0	0	0	0	50	50	55	100	90	0	0	0	0	0	0	0	65	60	65	85	85	45	0	0
25	0	0	0	0	0	50	50	55	100	95	100	70	100	65	35	30	65	60	65	85	85	50	0	0	0
20	0	0	0	0	0	50	55	55	80	100	100	75	100	70	40	30	65	60	65	85	90	55	0	0	0
15	0	0	0	0	0	50	60	60	85	75	75	80	45	75	45	30	65	60	65	90	95	60	0	0	0
10	0	0	0	0	0	50	65	65	90	80	80	85	50	80	50	35	70	65	70	95	100	65	0	0	0
5	0	0	0	0	0	35	35	70	95	85	85	90	55	85	55	40	75	70	70	100	100	70	25	15	15
0	0	0	0	10	0	40	40	40	35	90	90	95	60	90	60	30	60	60	60	80	80	75	30	20	20

Figure 29. Optimal Strategy, Net Load Profile 2, Scenario 4

State of Charge (kWh)	Time of Day																							
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	80	80	0	0	0
95	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	65	85	85	0	0	0
90	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	65	0	65	90	90	0	0	0	0
85	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	70	0	70	95	95	0	0	0	0
80	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	70	0	75	100	100	0	0	0	0
75	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	75	0	80	100	100	0	0	0	0
70	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	80	0	85	100	100	0	0	0	0
65	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	85	95	90	100	100	75	0	0	0
60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	100	95	100	100	80	0	0
55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	95	95	100	100	100	85	0	0	0
50	0	0	0	0	0	0	0	0	0	0	0	0	0	0	70	0	100	100	100	100	90	0	0	0
45	0	0	0	0	0	0	0	0	0	0	0	0	0	80	75	0	100	100	100	100	95	0	0	0
40	0	0	0	0	0	0	0	0	0	80	0	0	85	80	0	100	75	100	100	85	100	0	0	0
35	0	0	0	0	0	95	100	95	0	95	85	85	0	90	85	0	100	80	100	100	90	100	0	0
30	0	0	0	0	0	100	100	100	100	100	90	0	0	0	0	0	100	85	100	100	95	100	0	0
25	0	0	0	0	0	100	100	100	100	100	95	95	100	100	95	65	100	90	100	100	100	100	0	0
20	0	0	0	0	0	100	100	100	100	100	100	100	100	100	100	70	100	95	100	100	100	100	0	0
15	0	0	0	0	0	100	100	100	100	100	100	100	100	100	100	75	100	100	100	100	100	100	0	0
10	0	0	0	0	0	100	100	100	100	100	100	100	100	100	100	80	100	100	100	95	100	100	0	90
5	0	0	0	100	0	100	100	100	100	100	100	85	85	100	100	85	100	100	100	100	100	100	100	95
0	90	0	0	100	100	100	100	100	100	100	100	90	90	100	100	90	60	100	100	100	100	100	100	100

Figure 30. Optimal Strategy, Net Load Profile 2, Scenario 5

State of Charge (kWh)	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
100	0	0	0	0	0	0	45	45	45	45	30	30	30	30	30	30	30	60	60	80	80	40	0	0
95	0	0	0	0	0	45	45	45	45	30	30	30	30	0	30	30	30	60	60	80	80	40	0	0
90	0	0	0	0	0	40	45	45	45	30	30	30	30	0	30	30	30	60	60	80	80	40	0	0
85	0	0	0	0	0	40	40	40	40	30	30	30	30	0	30	30	30	60	60	80	80	40	0	0
80	0	0	0	0	0	40	40	40	40	30	30	30	30	0	30	30	30	60	60	80	80	40	0	0
75	0	0	0	0	0	40	40	40	40	30	30	30	30	0	30	30	30	60	60	80	80	40	0	0
70	0	0	0	0	0	40	40	40	40	30	30	30	30	0	0	30	30	60	60	80	80	40	0	0
65	0	0	0	0	0	40	40	40	40	30	30	30	30	0	0	0	30	60	60	80	80	40	0	0
60	0	0	0	0	0	40	40	40	40	30	30	30	30	0	0	0	30	60	60	80	80	40	0	0
55	0	0	0	0	0	40	40	40	40	30	30	30	30	0	30	30	30	60	60	80	80	40	0	0
50	0	0	0	0	0	40	40	40	40	30	30	30	30	30	30	30	30	60	60	80	80	40	0	0
45	0	0	0	0	0	40	40	40	40	30	30	30	30	30	30	30	30	60	60	80	80	40	0	0
40	0	0	0	0	0	0	40	40	40	30	30	30	30	30	30	30	30	60	60	80	80	40	0	0
35	0	0	0	0	0	40	40	40	40	30	30	30	30	30	30	30	30	60	60	80	80	40	0	0
30	0	0	0	0	0	40	40	40	45	30	30	30	30	30	30	30	30	60	60	80	80	40	0	0
25	0	0	0	0	0	40	40	45	45	30	30	30	30	30	30	30	30	60	60	80	80	45	0	0
20	0	0	0	0	0	40	40	45	45	30	30	30	35	30	30	30	65	60	65	85	85	50	0	0
15	0	0	0	0	0	35	40	40	45	30	30	30	35	35	30	30	65	60	65	85	85	55	0	0
10	0	0	0	0	0	35	30	30	40	30	30	30	35	35	30	30	65	60	65	85	90	60	0	10
5	0	0	0	0	0	35	35	35	30	30	30	30	30	35	30	30	65	60	65	90	95	65	25	10
0	5	0	0	10	5	40	40	40	35	30	30	30	30	30	30	30	60	60	60	80	80	70	30	15

Time of Day

Figure 31. Optimal Strategy, Net Load Profile 2, Scenario 6

State of Charge (kWh)	Time of Day																							
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
100	0	0	0	0	0	45	45	45	45	45	30	30	30	35	30	30	65	60	65	80	80	45	0	0
95	0	0	0	0	0	45	45	45	45	45	30	30	35	35	35	30	65	60	65	80	80	45	0	0
90	0	0	0	0	0	45	45	45	45	45	30	30	30	35	30	30	65	60	60	80	80	40	0	0
85	0	0	0	0	0	45	45	45	45	45	30	30	30	30	30	30	60	60	60	80	80	40	0	0
80	0	0	0	0	0	45	45	45	45	45	30	30	30	0	30	30	60	60	60	80	80	40	0	0
75	0	0	0	0	0	45	45	45	45	45	30	30	30	0	30	30	60	60	60	80	80	40	0	0
70	0	0	0	0	0	45	45	45	45	45	30	30	30	0	30	30	60	60	60	80	80	40	0	0
65	0	0	0	0	0	45	45	45	45	45	30	30	30	0	30	30	60	60	60	80	80	40	0	0
60	0	0	0	0	0	40	45	45	45	45	30	30	30	0	30	30	60	60	60	80	80	40	0	0
55	0	0	0	0	0	40	45	45	45	45	30	30	30	30	30	30	60	60	60	80	80	40	0	0
50	0	0	0	0	0	40	45	45	45	45	30	30	30	30	30	30	60	60	60	80	80	40	0	0
45	0	0	0	0	0	40	45	45	45	45	30	30	30	30	30	30	60	60	60	80	80	40	0	0
40	0	0	0	0	0	40	45	45	45	45	30	30	30	30	30	30	60	60	60	80	80	40	0	0
35	0	0	0	0	0	40	45	45	45	45	30	30	30	30	30	30	60	60	60	80	80	40	0	0
30	0	0	0	0	0	40	45	45	45	45	30	30	30	30	30	30	60	60	60	80	80	45	0	0
25	0	0	0	0	0	40	45	45	45	45	30	30	35	30	30	30	65	60	65	80	80	45	0	0
20	0	0	0	0	0	40	45	45	45	45	30	30	35	35	30	30	65	60	65	85	85	50	0	0
15	0	0	0	0	0	40	40	45	45	45	30	35	35	35	30	30	65	60	65	85	85	55	0	0
10	0	0	0	0	0	35	40	40	45	45	30	35	35	35	35	35	65	65	70	85	90	60	0	10
5	0	0	0	0	0	35	35	35	35	35	30	30	35	35	35	35	70	60	70	90	95	65	25	15
0	5	0	0	10	5	40	40	40	35	30	30	30	30	30	30	30	60	60	60	80	80	70	30	15

Figure 32. Optimal Strategy, Net Load Profile 2, Scenario 7

State of Charge (kWh)	Time of Day																							
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
100	0	0	0	0	0	0	40	40	40	40	30	30	0	0	0	0	0	60	60	60	80	80	40	0
95	0	0	0	0	0	40	40	40	45	45	30	0	0	0	0	0	0	60	60	60	80	80	40	0
90	0	0	0	0	0	45	45	45	45	45	0	0	0	0	0	0	0	65	60	65	80	80	45	0
85	0	0	0	0	0	45	45	45	45	45	35	35	0	35	0	0	0	65	60	65	80	80	45	0
80	0	0	0	0	0	45	45	45	45	50	40	35	0	35	0	0	0	65	60	65	80	80	45	0
75	0	0	0	0	0	45	45	45	45	50	45	40	35	40	35	0	0	65	60	65	80	80	45	0
70	0	0	0	0	0	45	45	50	55	50	45	35	45	35	0	0	0	65	60	65	80	80	45	0
65	0	0	0	0	0	45	45	50	55	30	30	0	0	0	0	0	0	65	60	65	80	80	45	0
60	0	0	0	0	0	45	45	50	50	30	0	0	0	0	0	0	0	65	60	65	80	80	45	0
55	0	0	0	0	0	45	50	50	50	35	35	35	0	35	0	0	0	65	60	65	85	80	45	0
50	0	0	0	0	0	45	50	50	50	40	40	35	0	35	0	0	0	65	60	65	85	85	45	0
45	0	0	0	0	0	45	45	50	50	45	45	40	40	40	35	0	0	65	60	65	80	80	45	0
40	0	0	0	0	0	0	0	50	55	50	50	45	45	45	35	30	65	60	65	80	80	45	0	
35	0	0	0	0	0	45	45	50	0	55	55	30	0	0	30	30	65	60	65	80	80	45	0	
30	0	0	0	0	0	45	45	50	50	30	0	0	0	0	0	0	65	60	65	80	80	45	0	
25	0	0	0	0	0	45	45	50	50	35	35	35	35	35	35	30	65	60	65	85	85	50	0	
20	0	0	0	0	0	45	50	50	50	40	40	40	40	40	40	30	65	65	70	90	85	55	0	
15	0	0	0	0	0	45	50	50	50	45	45	45	40	45	45	35	70	60	70	95	90	60	15	
10	0	0	0	0	0	40	45	45	50	50	50	50	45	50	50	40	75	65	75	100	95	65	0	
5	0	0	0	0	0	35	35	35	35	55	55	55	55	55	55	45	70	70	80	95	100	70	25	
0	5	0	0	10	5	40	40	40	35	30	30	30	30	30	30	30	60	60	60	80	80	75	30	

Figure 33. Optimal Strategy, Net Load Profile 2, Scenario 8



State of Charge (kWh)	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	
100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	60	60	60	80	80	0	0	0
95	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	60	60	60	80	85	0	0	0
90	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	65	60	65	85	85	0	0	0
85	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	65	60	65	85	90	0	0	0
80	0	0	0	0	0	0	0	0	0	40	0	0	0	0	0	0	0	65	65	70	90	95	0	0	0
75	0	0	0	0	0	0	0	0	0	45	0	0	0	50	0	0	0	70	60	70	95	100	0	0	0
70	0	0	0	0	0	0	0	0	0	50	0	0	0	55	0	0	0	75	65	75	100	100	0	0	0
65	0	0	0	0	0	0	0	0	0	55	0	0	0	0	0	0	0	70	70	80	100	100	70	0	0
60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	75	75	85	100	100	75	0	0
55	0	0	0	0	0	0	0	0	0	65	0	0	0	0	0	0	0	80	65	90	100	100	80	0	0
50	0	0	0	0	0	0	0	0	0	70	40	0	0	50	50	0	85	65	95	100	100	85	0	0	0
45	0	0	0	0	0	0	0	0	0	75	45	0	0	55	55	0	90	65	100	100	100	90	0	0	0
40	0	0	0	0	0	0	0	0	0	80	50	0	0	60	60	0	95	65	100	100	85	95	0	0	0
35	0	0	0	0	0	85	100	0	0	85	55	0	0	65	65	0	100	65	100	100	85	100	0	0	0
30	0	0	0	0	0	90	100	95	100	90	0	0	0	0	0	0	100	70	100	100	90	100	0	0	0
25	0	0	0	0	0	95	100	100	100	95	65	35	100	75	40	40	70	65	100	100	95	80	0	0	0
20	0	0	0	0	0	100	60	100	100	100	70	40	100	80	45	45	70	70	100	100	100	85	0	0	0
15	0	0	0	0	0	100	65	100	85	100	75	45	100	85	50	50	70	75	100	100	100	90	0	0	0
10	0	0	0	0	0	100	70	100	90	100	80	50	100	90	55	55	70	80	100	100	100	95	0	90	0
5	0	0	0	90	0	100	75	100	95	100	85	55	85	95	60	60	70	85	100	100	100	100	100	95	0
0	90	0	0	95	90	100	80	100	100	90	90	60	30	100	65	30	60	90	100	80	80	100	100	100	100

Time of Day

Figure 34. Optimal Strategy, Net Load Profile 2, Scenario 9